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AN INVENTORY MODEL FOR PERISHABLE ITEMS ADDRESSING SIX DEMAND TYPES, NON-LINEAR HOLDING COSTS, AND INVESTMENT IN GREEN TECHNOLOGY.

Rajarajeswari V PG and Research Department of Mathematics, M.V.Muthiah Government Arts College for women, Dindigul, 624001, Tamilnadu, India. E-mail(s): profrajarajeswari@gmail.com
Annadurai K PG and Research Department of Mathematics, M.V.Muthiah Government Arts College for women, Dindigul, 624001, Tamilnadu, India. E-mail(s): drkannadurai@gmail.com

Abstract
Consumers are more health-conscious than ever, and there has been a sharp rise in the demand for fresh goods. In this situation, preventing the associated losses from perishable goods deterioration requires an effective and efficient inventory management system. In addition, a number of factors, including price, stock levels, and freshness state, affect product demand. As a result, this study creates an inventory model for perishable goods that is limited by deterioration in both physical and freshness conditions. Green technologies and preservation are also employed to slow down the rate of deterioration and carbon emissions, correspondingly. Perishable goods demand is influenced by a number of variables, including price, quantity in stock, and freshness. Six price-dependent demand functions (linear, isoelastic, exponential, logit, logarithmic, and polynomial) are used in relation to price. This inventory model also accounts for the cost of deterioration and the expiration
date, because it works with perishable items that eventually deteriorate. In addition, a quadratic function of time is used to model the holding cost. In order to maximize total profit per unit of time, the suggested inventory model simultaneously determines the best price, the replenishment cycle time, and the order quantity. This inventory model is widely applicable because it can be used in a variety of contexts, including the production of food products (such as milk, vegetables, and meat), living things, and decorative flowers, among others. Use of this model is demonstrated with a few numerical examples. A sensitivity analysis is then carried out, and various managerial insights are offered.

Keywords:
Green technology, carbon tax policy, six different price-dependent demands, perishable items.

1 Introduction

Worldwide, inventory management is a crucial function for businesses. In order to prevent overstock and/or stockouts, it aims to maintain control over the materials from acquisition through sales-related decision-making (how much and when to buy items). Annadurai and rajarajeswari [1] addressed an integrated mixture of distribution model for environmental cost with fuzzy demand. Yavari et al. [21] indicated that one of the difficulties in inventory management is the perishability of many products, which means that their quality and freshness deteriorate with time and that they cannot be sold after their expiration date. As stated by Tirkolaee et al.[15], organisms, decorative flowers, and food items (such as milk, vegetables, and meat) all have a built-in perishability. These authors also mentioned how important it is for producers and buyers to have a window of time between preparing and selling perishable goods. Over the past few decades, supply chain managers have placed a strong emphasis on reducing the energy consumption and greenhouse gas (GHG) emissions corresponding with their production and logistics systems. This interest arose as a result of social pressure and consumer awareness of the importance of sustainability to their communities, which prompted governments to enact laws that took this viewpoint into consideration in an effort to improve the preservation of natural resources and reduce the negative environmental effects of product manufacture, use, and disposal. Energy efficiency has the potential to increase supply chain efficiency. Supply chain management is unique in that it incorporates environmental considerations alongside the traditional economic focus, without bringing in the concept of carbon emissions. Supply chain managers can lessen their impact on the environment by using energy more wisely and producing fewer emissions. Long-term cost savings and enhanced environmental performance are possible outcomes. Energy storage technologies and renewable energy sources can also lower emissions and boost efficiency. While lowering energy consumption and carbon emissions in conventional manufacturing is challenging, it is achievable with production rate management.

Novelty of the study

As closing the stock cycle with zero inventory is the best course of action when handling perishable goods, this research project creates an inventory model for perishable goods that have zero inventory at the conclusion of the stock cycle. While the demand for these perishable goods is influenced by a number of factors, including price, quantity of on-hand stock, and freshness condition, they are also susceptible to physical deterioration and freshness degradation over time. Six different price-dependent demand functions-linear, isoelastic, exponential, logit, logarithmic, and polynomial-are taken into consideration for the price-related demand. The perishable item's expiration date, salvage value, and deterioration cost are also taken into account by the inventory model. Furthermore, it is believed that the holding cost has a quadratic function of time and is nonlinear. The suggested inventory model simultaneously determines the best course of action for the quantity ordered, the price, and the replenishment cycle time, all of which work together to maximize the overall profit per unit of time. Prioritizing environmental preservation and reducing waste and carbon emissions during supply chain operations, including production, rework, transportation, storage, and deterioration, is crucial. This can be achieved through strategic implementation of carbon tax policies. There is still work to be done on applying learning concepts, enhancing inventory quality, assessing the impact of inflation under investment, and protecting products and energy flexibility using preservation technology. Because of this, it hasn't happened, which is what makes this research paper so special. Therefore, it is crucial to look at how decision-makers Six different price-dependent demand functions, waste management, perishable goods, green technology, and preservation technology into their supply chain inventory system in an uncertain environment in order to achieve environmental and economic sustainability.

Structure of this study

The remaining portions of this research are divided into the following sections. In section 2 includes a literature review to provide motivation for the current work. Section 3 contains notations and basic assumptions required for modeling purposes. In section 4 the inventory model with price, stock, and time-dependent demand with nonlinear holding cost is developed. Section 5 develops the solution procedure to determine the optimal solution. Section 6 presents the solution to six different price-dependent demand functions and solves some numerical examples. A sensitivity analysis with managerial insights are proposed in section 7. Finally, Section 8 provides the conclusions and outlines several areas for further research.

2 Literature review

In this section, we have to discussed the literature review related in different direction (1) Inventory model with price,stock, and age- dependent demand, with zero inventory at the end of the cycle (2) Inventory models with carbon emissions & energy usage.

2.1 Inventory model with price,stock, and age- dependent demand, with zero inventory at the end of the cycle

Pal et al. [12] addressed a production-inventory model for deteriorating products when the production cost depends on both production order quantity and production rate, given that the inherent perishability can occur immediately. Subsequently, while taking into account several just-in-time deliveries, Mashud et al. [5] identified the best replenishment strategy for deteriorating goods for the traditional newsboy inventory problem. Furthermore, Mashud et al. [2] developed an inventory model for deteriorating products that determines the ideal values for price, green investment cost, and replenishment time. Additionally, some products have a non-instantaneous intrinsic perishability. Mashud et al. [4] created an inventory model for non-instantaneous deteriorating items that optimises cycle time, price, preservation technology, and credit financing. Mashud et al. [3] proposed an inventory model for non-instantaneous deteriorating goods, determining cycle length, price, and preservation costs. Hasan et al. [22] proposed a non-instantaneous inventory model for agricultural goods that accounts for their perishable nature. This model identifies optimal inventory pricing and timing policies. Consumers are becoming more healthconscious, leading to an increase in demand for fresh items. Consumers prefer to purchase fresh goods with extended shelf life. Companies should carefully manage and control their fresh inventory in warehouses. RFID smart tags are commonly used to track the freshness and quality of perishable items in warehouses, reducing the risk of selling expired items. Herbon et al. [6], Herbon et al. [7], and Herbon and Ceder [8] studied the impact of using the timetemperature-indicator (TTI) to provide online expiration dates for various items. Displaying large quantities on shelves can encourage consumers to buy more products. Retailers can increase profits by making their products more available. Perishable items, which degrade over time, may not be as appealing or sellable at the end of their shelf life. Lower prices lead to increased demand, while higher prices lead to decreased demand. Price, stock availability, time, and shelf-life are key factors influencing demand for perishable products and should be taken into account when creating inventory models. The literature on inventory models with time, price, and stock-dependent demand is extensive. Mashud et al. [9] developed a price-sensitive inventory model based on an exponential or isoelastic price-dependent demand. Avinadav et al. [34] proposed two inventory models based on the assumption that demand is influenced by both price and stock age, resulting in optimal pricing and inventory policies.

The first inventory model assumes multiplicative demand, whereas the second assumes additive demand. Qin et al. [37] developed an inventory model to determine pricing and policies for perishable items, taking into account stockdependent demand and quality degradation. Later, Chen et al. [33] determined the optimal inventory policy and shelf-space size for fresh products, taking into account expiration time and demand rates based on freshness and stock levels. Font et al. [16] resolved Chen et al.’s [33] inventory model by incorporating the demand rate as a function of price, stock, and age. Dobson et al. [17] developed an EOQ inventory model for a perishable item with a fixed shelf-life. The model assumes that demand decreases linearly with stock age. Herbon and Khmelnitsky [10] developed an inventory model to determine the optimal replenishment time and price based on the demand rate, which varies with both time and price. Banerjee and Agrawal [32] proposed optimal discounting and ordering policies for deteriorating products, with a demand rate based on price and freshness. Hsieh and Dye [35] proposed an optimal pricing policy for deteriorating goods, taking into account the impact of reference prices and the assumption that stocks stimulate demand. Li and Teng [26] developed pricing strategies for perishable items based on reference price, inventory, and freshness. Recently, Agi and Soni [23] created an inventory model that optimises pricing and inventory management for perishable items with stock, age, and price-dependent demand, allowing for surplus inventory at the end of the cycle. These authors stated that finishing the inventory cycle period with a positive inventory level results in a benefit because the demand increases when large quantities are ordered and exhibited in the shelf space. Keeping perishable products at the end of their cycle is not ideal as they cannot be stored for the next inventory cycle. Therefore, these must be sold at a salvage price. Inventory models typically assume a constant holding cost, but factors such as storage location can cause this cost to vary. Longer storage periods often lead to higher holding costs due to the need for more expensive warehouse facilities. Longterm storage of fresh products requires refrigeration and specific conditions to prevent damage. Alfares and Ghaithan [18] conducted a comprehensive review of the EOQ and EPQ inventory models with variable holding costs. Specifically, variable holding costs into three types: time-dependent, stock-dependent, and multiple dependence cost variability. There are several types of holding cost functions: constant, linear, nonlinear, step, and general. There are several functions available to model variable time-dependent holding costs. Valliathal and Uthayakumar [24] used a quadratic holding cost function to create two EPQ inventory models for deteriorating items. The holding cost was nonlinear and time-dependent. Years later, Pal et al. [13] studied the single-period newsvendor model and found that the optimal lot size for customers who baulk is determined by a nonlinear holding cost that varies with lot size and stock level. Tripathi and Mishra [27] studied two inventory models with time-varying holding costs to determine the optimal order quantity and

replenishment cycle time. The first inventory model has a linearly time-dependent holding cost, while the second has a quadratic time dependence for carrying inventory costs.
Sivashankari [14] compared three EPQ inventory models with the assumption that carrying inventory costs are constant, linear, or quadratic over time.

2.2 Inventory models with carbon emissions and energy usage

The role of renewable energy is critical in developing sustainable supply chains. Sustainability consists of three fundamental pillars: economic, environmental, and social. Renewable energy impacts all three pillars of economic development by making traditional energy more expensive for companies. Developing countries have implemented emission-reduction policies (e.g., cap and trade) and advanced technology to reduce carbon dioxide emissions, as rising demand leads to higher emissions. The authors explored how government regulation can reduce carbon emissions and energy use in production systems. Feng et al. [20] proposed a supply chain inventory model that integrates environmental, social, and economic considerations into business operations to enhance sustainability. Ahi and Searcy [11] conducted extensive research on the incorporation of sustainability concepts into the supply chain. Sarkar et al. [30] proposed a sustainable inventory model for multi-items with imperfect production processes and optimal energy consumption. Mashud et al. [25] developed a sustainable inventory approach with controllable carbon emissions. Thomas and Mishra [36] developed a circular economy inventory model to reduce waste and pollution through 3D printing and other emission-reducing mechanisms. Using waste and carbon reduction technologies led to significant profit increases in the plastic reforming industry. Ruidas et al. [29] proposed a sustainable economic production quantity model for green degree products with green subsidies. Higher subsidy intensity improves product greenness, while investing in green innovation and emission reduction technology benefits both manufacturers and the environment. Jauhari [19] developed a supply chain system for imperfect products that prioritises optimum energy consumption and controllable production rates. Ruidas et al. [28] proposed a production inventory model with price and green degree-dependent demand, accounting for cap-and-trade policies. The joint investment in Green Innovation (GI) and Emission Reduction Technology (ERT) benefits both the green product manufacturer and the environment. Sarkar et al.[31] developed a sustainable inventory system for substitutable products under a dual channel policy and a fully controlled emission production system. They saw that investments made in green projects benefit the environment.

3 Notation and Assumptions

3.1 Notation

The following notations are used to develop the inventory model with age, price, and stock dependent demand and zero inventory at cycle's end with carbon emission. In order to have a standard notation, the symbols of Agi and Soni [23] are used, and a few more symbols are defined here.

A Cost of salvaging a degraded item
P Purchase price (unit/\$)
D Cost of deterioration (in units or \$)
H Storage fee (monetary sign, unit, or time unit)
H_a Holding cost (\$/$unit/unitoftime^2$)
H_b Holding cost (\$/$unit/unitoftime^3$)
O Ordering cost (\$/$percycle$)
N The item's shelf life, after which any leftover quantity is removed right away from storage (measured in units of time)
W_0 Maximum unit of shelf space.(units) b_0 Salvage coefficient($0 \leq \eta \leq 1$)
θ_0 Inventory deterioration rate ($0 \leq \theta \leq 1$)
ω_0 Sensitivity parameter to the current level of stock
a_0 Scale parameter for the part of price-dependent demand
b_0 Sensitivity parameter for the part of price-dependent demand
$D_0(p)$ Price-dependent demand for the item, which can be linear, isoelastic, exponential, logit, logarithmic, or polynomial function (units/unit of

time)

$i_0(T)$ Stock level at time T (units)

$d(p_0, i_0(T), T)$ Price, stock, and age-dependent demand for the item at time T (units/unit of time)

$h(T)$ Quadratic holding cost function

$\pi_c(p,t)$ Total profit per cycle ($\$/unit$)

$\pi(p,t)$ Total profit per unit of time ($\$/unitoftime$)

T Age of the stock, which is the time passed since the last replenishment (unit of time)

P Product selling price ($\$/unit$)

T Age of the stock, which is the time passed since the last replenishment (unit of time)

q Order quantity (units)

N_0 Fixed transportation cost ($\$/delivery$)

N_1 Variable transportation cost ($\$/delivery$)

F_1 Fuel consumption of an empty truck (litre/km)

F_2 Fuel consumption per ton q (litre/km)

v_c Distance travelled from producer to retailer (km)

ϵ Investment in green technology ($\$/unit/month$)

c_e Carbon emission caused by holding inventory ($tonCO_2/unit$)

τ Cost for emission of carbon from vehicles ($\$/km$)

d_t Cost for emission of carbon from transporting items ($\$/unit/km$)

c_d Carbon emission caused by retailer's deteriorating inventory ($tonCO_2/unit$)

λ_0 Proportion of carbon emission after investment in green ($0 < \lambda_0 < 1$)

3.2 Assumptions

The inventory model is based on the following assumptions:

1. Over time, the item in storage is susceptible to two types of degradation: freshness degradation and physical degradation at a constant rate.
2. The product has a set, finite shelf life beyond which it can no longer be sold.
3. Price, the quantity of the item in stock at any given time, and freshness all affect demand. Six distinct price-dependent demand functions are used to account for price-dependent demand: logit, logarithmic, polynomial, exponential, linear, and isoelastic.
4. At the conclusion of the cycle, there are none left.
5. The holding cost is nonlinear and is represented by a time-dependent quadratic function ($H + H_1T + H_2T^2$).
6. For the items that deteriorated over the course of the inventory cycle, the salvage value and deterioration cost are considered.
7. The planning time horizon is infinite.Since there is no lead time, there is an instantaneous replenishment rate.
8. The item is fresh at the start of the stock period ($T = 0$) and its age has no bearing on demand. With time, the product loses its freshness, which lowers demand for it.
9. Given that the item becomes unsalable after its expiration date ($t \leq N$), the stock period (t) cannot be longer than its shelf life (N).
10. Carbon emissions stem from a multitude of supply chain system operations, including setup, production, rework, transportation, inventory holding, waste management, and deterioration.
11. Decision-makers combine a carbon tax policy with a creative investment in green technology to lower the rate of carbon emissions.

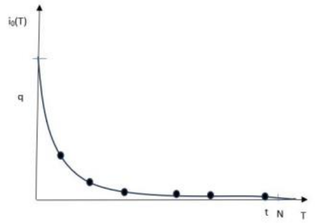

Fig. 1 *Graphical representation of the inventory level over time*

4 Mathematical formulation

4.1 Inventory model with price, stock, and ageDependent Demand, with Zero Inventory at the End of the Cycle

The problem being studied is described below. A business oversees a product that is naturally perishable and is susceptible to deterioration in terms of both freshness and physical state. Furthermore, it is well known that the product has a finite shelf life, after which it can no longer be sold. Because of this, the stock cycle cannot last longer than the item's shelf life. Because of the nature of the product, demand is influenced by its price, availability, and freshness. Conversely, the goal is to finish the inventory cycle with zero inventory. For the deteriorated items over the course of the inventory period, the salvage value and deterioration cost are also taken into account. The inventory level's behavior over time is depicted in Figure 1. The retailer receives a lot size of q units at the start of the stock cycle ($T = 0$), and the inventory level starts to decline right away as a result of both deterioration and demand. This process continues until the stock level hits zero units at $t = T$.

The on-hand stock degrades at a steady rate (θ_0) and gradually loses freshness over the course of the stock cycle $[0, t]$. The following function indicates how price, stock, and age affect demand:

$$d(p_0, i_o(T), T) = \frac{N-T}{N} D_0(p) + \omega_0 i_o(T), \quad N > 0, \omega_0 \geq 0, T \leq N, \qquad (1)$$

the price-dependent demand component ($D_0(p)$) can be expressed as follows:

$$D_0(p) = a_0 - b_o p, \quad 0 \leq p \leq \frac{a}{b} \text{ (linear demand)}$$

$$D_0(p) = a_o p^{-b0}, \quad 0 \leq p \leq \infty \text{ (iso-elastic demand)}$$

$$D_0(p) = a_o e^{-b0p}, \quad 0 \leq p \leq \infty \text{ (exponental demand)}$$

$$D_0(p) = \frac{a_0}{1 + e^{b_0 p}}, \quad 0 \leq p \leq \infty \text{ (logit demand)}$$

$$D_0(p) = a_0 - b_o lnp, \quad 0 \leq p \leq e^{(a0/b0)} \text{ (logarithmic demand)}$$

$$D_0(p) = a_0 - b_o p^M, \quad 0 \leq p \leq \frac{a}{b}^{(1/M)} \text{ (polynomial demand)} \qquad (2)$$

This study's six price-dependent demand functions accurately model scenarios where demand rises as prices fall, and vice versa.

The differential equation below models the behavior of the on-hand inventory level $i_0(T)$, taking into account the assumptions mentioned above.

$$\frac{di_0(T)}{dT} = \frac{N-T}{N} D_0(p) - \omega_0 i_0(T) - \theta_0 i_0(T), \quad 0 \leq T \leq t, \qquad (3)$$

with the boundary condition

$$i_0(t) = 0. \qquad (4)$$

Solving differential equation (3) yields the inventory level $i_0(T)$ as shown below
:

$$i_0(T) = \frac{D_0(p)}{\omega_0 + \theta_0}\left(1 + \frac{1}{N(\omega_0+\theta_0)}\right)\left(e^{(\omega_0+\theta_0)(t-T)} - 1\right) - \frac{D_0(p)}{N(\omega_0+\theta_0)}\left(te^{(\omega_0+\theta_0)(t-T)} - T\right), \quad 0 \leq T \leq t. \qquad (5)$$

The order quantity q at $i_0(0)$ is expressed as follows :

$$q = \frac{D_0(p)}{\omega_0+\theta_0}\left(-1 - \frac{1}{N(\omega_0+\theta_0)}\right) + \frac{D_0(p)e^{(\omega_0+\theta_0)t}}{\omega_0+\theta_0}\left(1 + \frac{1}{N(\omega_0+\theta_0)} - \frac{t}{N}\right) \leq W_0.$$

(6)

The total costs calculated as ordering, holding, purchasing, deterioration, transportation and carbon emission. The ordering cost reflects the cost of placing an order. To calculate the holding cost, take the definite integral from zero to *t* of the product of the quadratic holding cost function $H(T)$ and the stock level function $i_0(T)$. The purchase cost is calculated by multiplying the unit purchase cost(C) by the order quantity (q). To calculate the deterioration cost, multiply the unit deterioration cost (CD) by the number of deteriorated units per cycle. The waste management process with consideration for the environment.

The detailed calculation of all components of the total cost function is mathematically presented below.

(1) Ordering cost (O) per cycle is

$$OC = O. \quad (7)$$

(2) The holding cost (HC) per cycle is

$$HC = \int_0^t H(T) i_0(T) dT = \int_0^T (H + H_1 T + H_2 T^2) i_0(T),$$

$$HC = H\Big[\frac{D_0(p)t}{\omega_0+\theta_0}\Big[-1+\frac{t}{2N}-\frac{1}{N(\omega_0+\theta_0)}\Big]+\Big[\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Big]\Big[\frac{D_0(p)}{(\omega_0+\theta_0)}\Big(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Big)\Big]\Big] + H_1\Big[\frac{D_0(p)t^2}{\omega_0+\theta_0}\Big[-\frac{1}{2}+\frac{t}{3N}-\frac{1}{2N(\omega_0+\theta_0)}\Big] + \Big[\frac{e^{(\omega_0+\theta_0)t}-(\omega_0+\theta_0)t-1}{(\omega_0+\theta_0)^2}\Big]\Big[\frac{D_0(p)}{(\omega_0+\theta_0)}\Big(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Big)\Big]\Big] + H_2\Big[\frac{D_0(p)t^3}{\omega_0+\theta_0}\Big[-\frac{1}{3}+\frac{t}{4N}-\frac{1}{3N(\omega_0+\theta_0)}\Big] + \Big[\frac{2e^{(\omega_0+\theta_0)t}-[t(\omega_0+\theta_0)(t(\omega_0+\theta_0)+2)]-2}{(\omega_0+\theta_0)^3}\Big]\Big[\frac{D_0(p)}{(\omega_0+\theta_0)}\Big(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Big)\Big]\Big]. \quad (8)$$

(3) Purchase cost (PC) per cycle is

$$PC = P\Big[\frac{D_0(p)}{(\omega_0+\theta_0)}\Big(-1-\frac{1}{N(\omega_0+\theta_0)}\Big)+\frac{D_0(p)e^{(\omega_0+\theta_0)t}}{\omega_0+\theta_0}\Big(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Big)\Big]. \quad (9)$$

(4) The deterioration cost (DC) per cycle is

$$DC = d(q - \int_0^T D_0(p)\Big(1-\frac{T}{N}\Big)+\omega_0 i_0(T)\Big),$$

$$DC = d\Big[\frac{D_0(p)}{\omega_0+\theta_0}\Big(-1-\frac{1}{N(\omega_0+\theta_0)}\Big)+\frac{D_0(p)e^{(\omega_0+\theta_0)t}}{\omega_0+\theta_0}\Big(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Big)\Big]-\Big[D_0(p)t\Big(1-\frac{t}{2N}+\frac{1}{N(\omega_0+\theta_0)}\Big)+\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Big(\frac{\omega_0 D_0(p)}{\omega_0+\theta_0}\Big)\Big(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Big)\Big]\Big].$$

(10)

(5) Transportation cost (T) is

$$T = N_0 + N_1\Big(2dF_1+\frac{dF_2 v_c}{\omega_0+\theta_0}\Big(e^{(\omega_0+\theta_0)t}-1\Big)\Big). \quad (11)$$

(6) Green technology cost is

$$GT = \epsilon\int_0^t e^{-kT}dT = \frac{\epsilon}{k}\Big(1-e^{-kT}\Big). \quad (12)$$

(7) Carbon emission cost from inventory sector energy and carbon emits because of transportation, carrying inventory, and deterioration. Therefore the total amount of carbon emission is

$$TE = \Big(2\tau+\frac{d_t v_c}{\omega_0+\theta_0}\Big(e^{(\omega_0+\theta_0)t}-1\Big)\frac{c_e v_c}{\omega_0+\theta_0}\Big)\Big(e^{(\omega_0+\theta_0)t}\Big(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Big)+\Big(c_d v_c\Big)\Big(e^{(\omega_0+\theta_0)t}\Big(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Big). \quad (13)$$

Now, the cost associated with the emissions and energy from the inventory sector under a carbon tax policy is $CE = t_e(TE)$

After investing in green technology, the total cost of carbon emissions

$$CE = (1-\lambda_0(1-e^{-\mu_0\epsilon}))(TE)$$

$$CE = t_e(1-\lambda_0(1-e^{-\mu_0\epsilon}))\Bigg[\Bigg(2\tau + \frac{d_t v_c}{\omega_0+\theta_0}\Bigg(e^{(\omega_0+\theta_0)t}-1\Bigg)\frac{c_e v_c}{\omega_0+\theta_0}\Bigg)$$
$$\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Bigg)$$
$$+\Bigg(c_d v_c\Bigg)\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Bigg)\Bigg]. \quad (14)$$

Therefore, the total cost per cycle is

TC = ordering cost + holding cost + purchase cost + deterioration cost+ transportation cost +green technology cost+ carbon emission cost. And, it is expressed as

$$TC(p,t,\epsilon) = O + \int_0^t H(T) i_0(T) dT$$
$$= H\Bigg[\frac{D_0(p)t}{\omega_0+\theta_0}\Bigg[-1+\frac{t}{2N}-\frac{1}{N(\omega_0+\theta_0)}\Bigg]+\Bigg[\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Bigg]\Bigg[\frac{D_0(p)}{(\omega_0+\theta_0)}$$
$$\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]\Bigg] + H_1\Bigg[\frac{D_0(p)t^2}{\omega_0+\theta_0}\Bigg[-\frac{1}{2}+\frac{t}{3N}-\frac{1}{2N(\omega_0+\theta_0)}\Bigg]$$
$$+\Bigg[\frac{e^{(\omega_0+\theta_0)t}-(\omega_0+\theta_0)t-1}{(\omega_0+\theta_0)^2}\Bigg]\Bigg[\frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]\Bigg] + H_2$$
$$\Bigg[\frac{D_0(p)t^3}{\omega_0+\theta_0}\Bigg[-\frac{1}{3}+\frac{t}{4N}-\frac{1}{3N(\omega_0+\theta_0)}\Bigg]+\Bigg[\frac{2e^{(\omega_0+\theta_0)t}-[t(\omega_0+\theta_0)(t(\omega_0+\theta_0)+2)]-2}{(\omega_0+\theta_0)^3}\Bigg]$$
$$\Bigg[\frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]\Bigg]$$
$$+P\Bigg[\frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(-1-\frac{1}{N(\omega_0+\theta_0)}\Bigg)+\frac{D_0(p)e^{(\omega_0+\theta_0)t}}{\omega_0+\theta_0}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]$$
$$+d\Bigg[\frac{D_0(p)}{\omega_0+\theta_0}\Bigg(-1-\frac{1}{N(\omega_0+\theta_0)}\Bigg)+\frac{D_0(p)e^{(\omega_0+\theta_0)t}}{\omega_0+\theta_0}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]$$
$$-\Bigg[D_0(p)t\Bigg(1-\frac{t}{2N}+\frac{1}{N(\omega_0+\theta_0)}\Bigg)+\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Bigg(\frac{\omega_0 D_0(p)}{\omega_0+\theta_0}\Bigg)$$
$$\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]\Bigg]+N_0+N_1\Bigg(2dF_1+\frac{dF_2 v_c}{\omega_0+\theta_0}\Bigg(e^{(\omega_0+\theta_0)t}-1\Bigg)\Bigg)$$
$$+\frac{\epsilon}{k}\Bigg(1-e^{-kT}\Bigg)+t_e(1-\lambda_0(1-e^{-\mu_0\epsilon}))\Bigg[\Bigg(2\tau+\frac{d_t v_c}{\omega_0+\theta_0}\Bigg(e^{(\omega_0+\theta_0)t}-1\Bigg)\frac{c_e v_c}{\omega_0+\theta_0}\Bigg)$$
$$\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Bigg)$$
$$+\Bigg(c_d v_c\Bigg)\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Bigg)\Bigg].$$

(15)

Hence, the total cost per unit of time is expressed as follows

$$TC(p,t,\epsilon) = \frac{1}{T}\Bigg(O + \int_0^t H(T) i_0(T) dT\Bigg)$$
$$= H\Bigg[\frac{D_0(p)t}{\omega_0+\theta_0}\Bigg[-1+\frac{t}{2N}-\frac{1}{N(\omega_0+\theta_0)}\Bigg]+\Bigg[\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Bigg]$$
$$\Bigg[\frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]\Bigg] + H_1\Bigg[\frac{D_0(p)t^2}{\omega_0+\theta_0}\Bigg[-\frac{1}{2}+\frac{t}{3N}-\frac{1}{2N(\omega_0+\theta_0)}\Bigg]$$
$$+\Bigg[\frac{e^{(\omega_0+\theta_0)t}-(\omega_0+\theta_0)t-1}{(\omega_0+\theta_0)^2}\Bigg]\Bigg[\frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]\Bigg]$$
$$+H_2\Bigg[\frac{D_0(p)t^3}{\omega_0+\theta_0}\Bigg[-\frac{1}{3}+\frac{t}{4N}-\frac{1}{3N(\omega_0+\theta_0)}\Bigg]$$
$$+\Bigg[\frac{2e^{(\omega_0+\theta_0)t}-[t(\omega_0+\theta_0)(t(\omega_0+\theta_0)+2)]-2}{(\omega_0+\theta_0)^3}\Bigg]\Bigg[\frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]\Bigg]$$
$$+P\Bigg[\frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(-1-\frac{1}{N(\omega_0+\theta_0)}\Bigg)+\frac{D_0(p)e^{(\omega_0+\theta_0)t}}{\omega_0+\theta_0}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]$$
$$+d\Bigg[\frac{D_0(p)}{\omega_0+\theta_0}\Bigg(-1-\frac{1}{N(\omega_0+\theta_0)}\Bigg)+\frac{D_0(p)e^{(\omega_0+\theta_0)t}}{\omega_0+\theta_0}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]$$
$$-\Bigg[D_0(p)t\Bigg(1-\frac{t}{2N}+\frac{1}{N(\omega_0+\theta_0)}\Bigg)+\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Bigg(\frac{\omega_0 D_0(p)}{\omega_0+\theta_0}\Bigg)$$
$$\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\Bigg)\Bigg]\Bigg]+N_0+N_1\Bigg(2dF_1+\frac{dF_2 v_c}{\omega_0+\theta_0}\Bigg(e^{(\omega_0+\theta_0)t}-1\Bigg)\Bigg)$$
$$+\frac{\epsilon}{k}\Bigg(1-e^{-kT}\Bigg)+t_e(1-\lambda_0(1-e^{-\mu_0\epsilon}))\Bigg[\Bigg(2\tau+\frac{d_t v_c}{\omega_0+\theta_0}\Bigg(e^{(\omega_0+\theta_0)t}-1\Bigg)\frac{c_e v_c}{\omega_0+\theta_0}\Bigg)$$
$$\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Bigg)$$
$$+\Bigg(c_d v_c\Bigg)\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Bigg)\Bigg]\Bigg).$$

(16)
The optimization problem is typically formulated as follows:
Minp,t,ϵTC(*p,t,ϵ*)
(17)

subject to $P \leq p \leq p_{\min}$ and $t \leq N$.

The total cost per unit of time function given in equation (16) illustrates a nonlinear relationship between selling price (*p*) and replenishment cycle time (*t*). As a result, these decision variables cannot be represented by a closed form. The optimal solution for p , T and ϵ is determined using traditional optimization criteria. The solution procedure is described in the following section.

5 Solution Procedure to Obtain the Optimal Solution

The goal is to find the optimal selling price (*p*) and replenishment cycle time (*t*) to minimize total costs. The cost per unit of time function *TC(p,t,ϵ)* is continuous and twice differentiable on the interval [0,∞], implying a global minimum on that interval.

To minimize the total cost per unit of time function *TC*(*p,t,ϵ*), the following conditions must be met.

$$\frac{\partial TC(p,t,\epsilon)}{\partial p} = 0 \quad (18)$$

$$\frac{\partial TC(p,t,\epsilon)}{\partial T} = 0, \quad (19)$$

$$\frac{\partial TC(p,t,\epsilon)}{\partial \epsilon} = 0. \quad (20)$$

Moreover, for the expected total cost per unit of time function *TC*(*p,t,ϵ*) to be convex, the sufficient conditions are given as follows:

$$\frac{\partial^2 TC(p,t,\epsilon)}{\partial p^2} \geq 0, \quad (21)$$

$$\frac{\partial^2 TC(p,t,\epsilon)}{\partial T^2} \geq 0, \quad (22)$$

$$\frac{\partial^2 TC(p,t,\epsilon)}{\partial \epsilon^2} \geq 0. \quad (23)$$

The nature of the Hessian matrix is determined by using

$$\begin{bmatrix} \frac{\partial^2 TC(p,t,\epsilon)}{\partial p^2} & \frac{\partial^2 TC(p,t,\epsilon)}{\partial p \partial T} & \frac{\partial^2 TC(p,t,\epsilon)}{\partial p \partial \epsilon} \\ \frac{\partial^2 TC(p,t,\epsilon)}{\partial T \partial p} & \frac{\partial^2 TC(p,t,\epsilon)}{\partial T^2} & \frac{\partial^2 TC(p,t,\epsilon)}{\partial T \partial \epsilon} \\ \frac{\partial^2 TC(p,t,\epsilon)}{\partial \epsilon \partial p} & \frac{\partial^2 TC(p,t,\epsilon)}{\partial \epsilon \partial T} & \frac{\partial^2 TC(p,t,\epsilon)}{\partial \epsilon^2} \end{bmatrix}.$$

The optimal solution is found by solving the first partial derivatives of the total cost per unit of time function in equation (16) with respect to p, T, and ϵ equating zero. If the solution (*p,T,ϵ*) meets the conditions in equations (18-23), it indicates that the function *TC*(*p,T,ϵ*) is strictly convex in decision variables and has a positive definite Hessian matrix. If true, the solution *TC*(*p,T,ϵ*) is optimal.

The selling price has a lower bound of p_{min} and the replenishment cycle time has a lower bound of N in the optimization problem given by equation (17). The demand component $D_0(p)$ that is correlated with price is represented by six distinct functions. The price p interval for isoelastic, exponential, and logit functions is [0,∞]. On the other hand, there is a minimum allowable value p_{min} for polynomial, logarithmic, and linear functions. For the linear, logarithmic, and polynomial cases, the lower bound p_{min} is (a/b), $e^{(a/b)}$, and $(a/b)^{(1/m)}$ respectively. When the selling price p is greater than the selling price p_{min}, the selling price solution for these demand functions is $p = p_{min}$. This makes it possible to prevent a price-dependent demand that is positive. While mathematically correct, it is not applicable to real-world situations the replenishment cycle time t lies between 0 and ∞. Because of the shelf life constraint, the replenishment cycle time solution is $t = N$ when the solution for the replenishment cycle time t is greater than N.

Algorithm

Algorithm for finding the best solution. Based on the previous section’s theoretical results, we propose the following algorithm (Algorithm 1).

Algorithm 1

1. Input the inventory parameters.
2. Calculate p_{min}.
3. Solve simultaneously equations (18),(19) and (20) to obtain the values for p , t and ϵ.
4. Proceed to step 5 if the conditions (18)–(23) are met, indicating that the solution is ideal. If not, the solutions are not workable and you should proceed to step 14.

5. If both $P \geq p \geq p_{min}$ and $t \geq N$ are satisfied, then set $p^* = p$ and $t^* = t$, and go to step 11. Else, go to step 6.
6. If both $p \geq P$ and $t \geq N$ are satisfied, then set $p^* = P$ and $t^* = t$, and go to step 11. Else, go to step 7.
7. If both $p \geq P$ and $t < N$ are satisfied, then set $p^* = P$ and $t^* = N$, and go to step 11. Else, go to step 8.
8. If both $p > p_{min}$ and $t \leq N$ are satisfied, then set $p^* = p_{min}$ and $t^* = t$, and go to step 11. Else, go to step 9.
9. If both $p \leq p_{min}$ and $t < N$ are satisfied, then set $p^* = p$ and $t^* = N$, and go to step 11. Else, go to step 10.
10. Set $p = p_{min}$ and $t^* = N$.
11. Calculate the lot size q^* with equation (6).
12. Compute the total profit per unit of time $TC^*(p^*,t^*,\epsilon)$ with equation (16).
13. Report the optimal solution $TC^*(p^*,t^*)$, p_*, t^*, $\epsilon^*$and q^*.
14. Stop.

6 Optimal inventory policy or six different price-dependent demands with carbon emission

6.1 Optimal inventory policy using the price-dependent linear demand function $D_0(p) = a_0 - b_0(p)$

The first partial derivative of $TC(p,t,\epsilon)$ with respect to p is

$$\frac{\partial TC(p,t,\epsilon)}{\partial p} = (-Hb_0)\left[\frac{1}{\omega_0+\theta_0}\left[-1+\frac{t}{2N}-\frac{1}{N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\right]\right.$$
$$\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]+(-H_1b_0)\left[\frac{t}{\omega_0+\theta_0}\left[-\frac{1}{2}+\frac{t}{3N}-\frac{1}{2N(\omega_0+\theta_0)}\right]\right.$$
$$\left.+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{e^{(\omega_0+\theta_0)t}-1}{(\omega_0+\theta_0)}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right]$$
$$+(-H_2b_0)\left[\frac{t^2}{\omega_0+\theta_0}\left[-\frac{1}{3}+\frac{t}{4N}-\frac{1}{3N(\omega_0+\theta_0)}\right]\right.$$
$$\left.+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{2e^{(\omega_0+\theta_0)t}-[t(\omega_0+\theta_0)(t(\omega_0+\theta_0)+2)]-2}{(\omega_0+\theta_0)^3}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right]$$
$$+(-b_0P)\left[\frac{1}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$-(-db_0)\left[\frac{1}{(\omega_0+\theta_0)t}\left(-1-\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$-1+\frac{t}{2N}+\frac{\omega_0}{(\omega_0+\theta_0)}\left(1-\frac{t}{2N}+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\left(\frac{\omega_0}{(\omega_0+\theta_0)t}\right)$$
$$\left.\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right]=0.$$

(24)

The first partial derivative of $TC(p,t,\epsilon)$ with respect to t is

$$\frac{\partial TC(p,t,\epsilon)}{\partial t} = -\Bigg[H\Bigg(\frac{D_0(p)}{2N(\omega_0+\theta_0)} + \frac{D_0(p)}{\omega_0+\theta_0}\Bigg(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)+1}{(\omega_0+\theta_0)t^2}\Bigg)\Bigg(1+\frac{1}{\omega_0+\theta_0}\Bigg)$$
$$-\frac{D_0(p)}{N(\omega_0+\theta_0)}e^{(\omega_0+\theta_0)t}\Bigg) + H_1\Bigg(-\frac{D_0(p)}{2(\omega_0+\theta_0)} + \frac{2D_0(p)t}{3N(\omega_0+\theta_0)}$$
$$-\frac{D_0(p)}{2n(\omega_0+\theta_0)^2} + \frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(\frac{\omega_0 te^{(\omega_0+\theta_0)t}+\theta_0 te^{(\omega_0+\theta_0)t}+1}{(\omega_0+\theta_0)^2t^2}\Bigg)\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg)$$
$$-\frac{D_0(p)}{N(\omega_0+\theta_0)}\Bigg(\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Bigg)\Bigg) + H_2\Bigg(-\frac{2D_0(p)t}{3(\omega_0+\theta_0)} + \frac{3D_0(p)t^2}{4N(\omega_0+\theta_0)} - \frac{2D_0(p)t}{3N(\omega_0+\theta_0)^2}$$
$$+\frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(\frac{-\theta_0^2t^2+2\omega_0\theta_0t^2-2\theta_0te^{(\omega_0+\theta_0)t}-\omega_0te^{(\omega_0+\theta_0)t}+2e^{(\omega_0+\theta_0)t}+\omega_0^2t^2-2}{(\omega_0+\theta_0)^3t^2}\Bigg)$$
$$\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) - \frac{D_0(p)}{N(\omega_0+\theta_0)}\Bigg(\frac{2e^{(\omega_0+\theta_0)t}-2t(\omega_0+\theta_0)-2}{(\omega_0+\theta_0)^2}\Bigg)\Bigg)$$
$$-\Bigg[P\Bigg(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) + \frac{D_0(p)}{\omega_0+\theta_0}\Bigg(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)}{t^2}\Bigg)$$
$$\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) - \frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Bigg)\Bigg]$$
$$-\Bigg[d\Bigg(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) + \frac{D_0(p)}{\omega_0+\theta_0}\Bigg(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)+1}{t^2}\Bigg)$$
$$\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) - \frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Bigg) + \frac{D_0(p)}{2N}\Bigg(1-\frac{\omega_0}{\omega_0+\theta_0}$$
$$-\frac{\omega_0 D_0(p)}{\omega_0+\theta_0}\Bigg(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)+1}{(\omega_0+\theta_0)t^2}\Bigg)\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) + \frac{\omega_0 D_0(p)}{(\omega_0+\theta_0)n}e^{(\omega_0+\theta_0)t}\Bigg)\Bigg]$$
$$+\frac{N_1 v_c e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t)-1}{\omega_0+\theta_0} + t_e(1-\lambda_0(1-e^{\mu_0\epsilon})) + \frac{d_t v_c}{\omega_0+\theta_0}\Bigg[\Bigg(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0)-1\Bigg)$$
$$\Bigg(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)\Bigg)\Bigg(\frac{1-e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}}{k}\Bigg)\Bigg] = 0.$$

(25)

The first partial derivative of $TC(p,t,\epsilon)$ with respect to ϵ is

$$\frac{\partial TC(p,t,\epsilon)}{\partial \epsilon} = t_e\lambda_0 e^{-\mu_0\epsilon}\ln(\epsilon)\Bigg[\Bigg(2\tau + \frac{d_t v_c}{\omega_0+\theta_0}\Bigg(e^{(\omega_0+\theta_0)t}-1\Bigg)\frac{c_e v_c}{\omega_0+\theta_0}\Bigg)$$
$$\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}-1}{k}\Bigg)$$
$$+\Bigg(c_d v_c\Bigg)\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}-1}{k}\Bigg)\Bigg] \quad (26)$$

6.1.1 Example 1

To illustrate a real-world situation, let's look at a store that offers freshly made goods. Assume that the demand for the fresh item is linear in its price-dependent component: $D_0(p) = 600 - 20p$. The shelf-life of the fresh item is $N = 1$ week. There is a minmum shelf space of $w_0 = 500$ units. The cost for placing an order to the supplier is $O = 250$ euros per order, the purchase cost is $P = 5$ euros per unit, and the sensitivity coefficient to the level of stock is $\omega_0 = 0.5$, and the stock deterioration rate is $\theta_0 = 0.05$. The aforementioned data were taken from Hasan et al. [22]. Additional data are still needed to solve the numerical examples. The values of the holding cost are $H = 1.75$ dollars per unit per week, $H_1 = 0.15$ dollars per unit per week^2, and $H_2 = 0.25$ dollars per unit per week^3. The deterioration cost of the item is $d = 2$ dollars per units and we have $b_0 = 0.8$ for the salvage coefficient. t must be less than one week because N equals one week for the replenishment cycle time. The lower bound for price in the price linear demand is given by $p_{min} = (a/b) = (600/20) = 30$. The following optimal solution for the inventory system is computed using the suggested algorithm $p^* = 16.69$ dollars per unit, $t^* = 0.4295$ weeks, $\epsilon^* = 0.2368$, $q^* = 93.4294$ units, and $TC^*(p^*,T^*,\epsilon^*) = 2048.903$ dollars per week. This solution satisfies all conditions for the optimality. It is ensured that the solution corresponds a global minimum (see Figure 2).

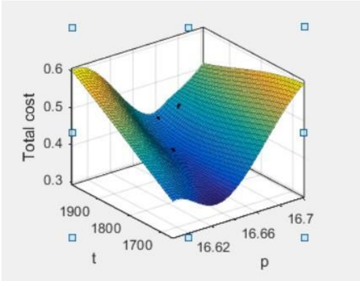

Fig. 2 *Convex property of the total cost TC(p,t,ϵ) when the pricedependent demand function is linear.*

6.2 The price-dependent isoelastic demand function with carbon emission $D_0(p) = a_0p^{-b_0}$

The first partial derivative of $TC(p,t,\epsilon)$ with respect to p is

$$\frac{\partial TC(p,t,\epsilon)}{\partial p} = \left(H\frac{a_0b_0}{p^{b_0+1}}\right)\left[\frac{1}{\omega_0+\theta_0}\left[-1+\frac{t}{2N}-\frac{1}{N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\right]\right.$$
$$\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]+\left(-H_1\frac{a_0b_0}{p^{b_0+1}}\right)\left[\frac{t}{\omega_0+\theta_0}\left[-\frac{1}{2}+\frac{t}{3N}-\frac{1}{2N(\omega_0+\theta_0)}\right]\right.$$
$$\left.+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{e^{(\omega_0+\theta_0)t}-1}{(\omega_0+\theta_0)}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right]$$
$$+\left(H_2\frac{a_0b_0}{p^{b_0+1}}\right)\left[\frac{t^2}{\omega_0+\theta_0}\left[-\frac{1}{3}+\frac{t}{4N}-\frac{1}{3N(\omega_0+\theta_0)}\right]\right.$$
$$\left.+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{2e^{(\omega_0+\theta_0)t}-[t(\omega_0+\theta_0)(t(\omega_0+\theta_0)+2)]-2}{(\omega_0+\theta_0)^3}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right.$$
$$+\left(-\frac{a_0b_0}{p^{b_0+1}}P\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$-\left(-d\frac{a_0b_0}{p^{b_0+1}}\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(-1-\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$-1+\frac{t}{2N}+\frac{\omega_0}{(\omega_0+\theta_0)}\left(1-\frac{t}{2N}+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\left(\frac{\omega_0}{(\omega_0+\theta_0)t}\right)$$
$$\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\Bigg]=0.$$

(27)

The first partial derivative of TC(p,t,ϵ) with respect to t is

$$\frac{\partial TC(p,t,\epsilon)}{\partial t} = -\Bigg[H\Bigg(\frac{D_0(p)}{2N(\omega_0+\theta_0)} + \frac{D_0(p)}{\omega_0+\theta_0}\Bigg(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)+1}{(\omega_0+\theta_0)t^2}\Bigg)\Bigg(1+\frac{1}{\omega_0+\theta_0}\Bigg)$$
$$-\frac{D_0(p)}{N(\omega_0+\theta_0)}e^{(\omega_0+\theta_0)t}\Bigg) + H_1\Bigg(-\frac{D_0(p)}{2(\omega_0+\theta_0)} + \frac{2D_0(p)t}{3N(\omega_0+\theta_0)}$$
$$-\frac{D_0(p)}{2n(\omega_0+\theta_0)^2} + \frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(\frac{\omega_0 te^{(\omega_0+\theta_0)t}+\theta_0 te^{(\omega_0+\theta_0)t}+1}{(\omega_0+\theta_0)^2t^2}\Bigg)\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg)$$
$$-\frac{D_0(p)}{N(\omega_0+\theta_0)}\Bigg(\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Bigg)\Bigg) + H_2\Bigg(-\frac{2D_0(p)t}{3(\omega_0+\theta_0)} + \frac{3D_0(p)t^2}{4N(\omega_0+\theta_0)} - \frac{2D_0(p)t}{3N(\omega_0+\theta_0)^2}$$
$$+\frac{D_0(p)}{(\omega_0+\theta_0)}\Bigg(\frac{-\theta_0^2t^2+2\omega_0\theta_0t^2-2\theta_0te^{(\omega_0+\theta_0)t}-\omega_0te^{(\omega_0+\theta_0)t}+2e^{(\omega_0+\theta_0)t}+\omega_0^2t^2-2}{(\omega_0+\theta_0)^3t^2}\Bigg)$$
$$\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) - \frac{D_0(p)}{N(\omega_0+\theta_0)}\Bigg(\frac{2e^{(\omega_0+\theta_0)t}-2t(\omega_0+\theta_0)-2}{(\omega_0+\theta_0)^2}\Bigg)\Bigg)$$
$$-\Bigg[P\Bigg(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) + \frac{D_0(p)}{\omega_0+\theta_0}\Bigg(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)}{t^2}\Bigg)$$
$$\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) - \frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Bigg)\Bigg]$$
$$-\Bigg[d\Bigg(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) + \frac{D_0(p)}{\omega_0+\theta_0}\Bigg(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)+1}{t^2}\Bigg)$$
$$\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) - \frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Bigg) + \frac{D_0(p)}{2N}\Bigg(1-\frac{\omega_0}{\omega_0+\theta_0}$$
$$-\frac{\omega_0 D_0(p)}{\omega_0+\theta_0}\Bigg(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)+1}{(\omega_0+\theta_0)t^2}\Bigg)\Bigg(1+\frac{1}{N(\omega_0+\theta_0)}\Bigg) + \frac{\omega_0 D_0(p)}{(\omega_0+\theta_0)n}e^{(\omega_0+\theta_0)t}\Bigg)\Bigg]$$
$$+\frac{N_1 v_c e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t)-1}{\omega_0+\theta_0} + t_e(1-\lambda_0(1-e^{\mu_0\epsilon})) + \frac{d_t v_c}{\omega_0+\theta_0}\Bigg[\Bigg(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0)-1\Bigg)$$
$$\Bigg(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)\Bigg)\Bigg(\frac{1-e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}}{k}\Bigg)\Bigg] = 0.$$

(28)

The first partial derivative of $TC(p,t,\epsilon)$ with respect to ϵ is

$$\frac{\partial TC(p,t,\epsilon)}{\partial \epsilon} = t_e\lambda_0 e^{-\mu_0\epsilon}\ln(\epsilon)\Bigg[\Bigg(2\tau + \frac{d_t v_c}{\omega_0+\theta_0}\Bigg(e^{(\omega_0+\theta_0)t}-1\Bigg)\frac{c_e v_c}{\omega_0+\theta_0}\Bigg)$$
$$\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}-1}{k}\Bigg) \qquad (29)$$
$$+\Bigg(c_d v_c\Bigg)\Bigg(e^{(\omega_0+\theta_0)t}\Bigg(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}-1}{k}\Bigg)\Bigg].$$

6.2.1 Example 2

Let's look at the identical data from Example 1. Assume for the moment that the price-dependent component of demand for the perishable commodity has the following isoelastic function: $D_0(p) = 30000p^{-1.4}$. By using the algorithm proposed, the following optimal solution for the inventory model is determined: $p^* = 17.47$ dollars per unit, $t^* = 0.29969$ weeks,
$\epsilon^* = 0.2209$, $q^* = 142.51$ units, and $TC^*(p^*,T^*,\epsilon^*) = 5265.004$ dollars per week. The total cost function is strictly convex and has a positive definite Hessian matrix; the Hessian determinant is less than zero. Thus, the solution is the optimal. It is ensured that the solution corresponds a global minimum (see Figure 3).

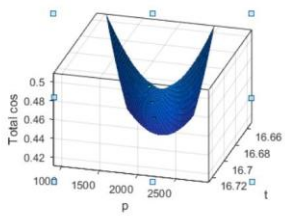

Fig. 3 *Convex property of the total cost TC(p,t,ϵ) when the pricedependent demand function is isoelastic.*

6.3 Optimal inventory policy using the price-dependent exponential demand function with carbon emission $D0(p) = a0e-b0p$

The first partial derivative of $TC(p,t,\epsilon)$ with respect to p is

$$\frac{\partial TC(p,t,\epsilon)}{\partial p} = \left(-H\frac{a_0b_0}{e^{b_0p}}\right)\left[\frac{1}{\omega_0+\theta_0}\left[-1+\frac{t}{2N}-\frac{1}{N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\right]\right.$$
$$\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]+\left(-H_1\frac{a_0b_0}{e^{b_0p}}\right)\left[\frac{t}{\omega_0+\theta_0}\left[-\frac{1}{2}+\frac{t}{3N}-\frac{1}{2N(\omega_0+\theta_0)}\right]\right.$$
$$\left.+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{e^{(\omega_0+\theta_0)t}-1}{(\omega_0+\theta_0)}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right]$$
$$+\left(H_2\frac{a_0b_0}{e^{b_0p}}\right)\left[\frac{t^2}{\omega_0+\theta_0}\left[-\frac{1}{3}+\frac{t}{4N}-\frac{1}{3N(\omega_0+\theta_0)}\right]\right.$$
$$\left.+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{2e^{(\omega_0+\theta_0)t}-[t(\omega_0+\theta_0)(t(\omega_0+\theta_0)+2)]-2}{(\omega_0+\theta_0)^3}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right.$$
$$+\left(-\frac{a_0b_0}{e^{b_0p}}\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$-\left(-d\frac{a_0b_0}{e^{b_0p}}\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(-1-\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$-1+\frac{t}{2N}+\frac{\omega_0}{(\omega_0+\theta_0)}\left(1-\frac{t}{2N}+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\left(\frac{\omega_0}{(\omega_0+\theta_0)t}\right)$$
$$\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\Bigg]=0.$$

(30)

$$\frac{\partial TC(p,t,\epsilon)}{\partial t} = -\Bigg[H\left(\frac{D_0(p)}{2N(\omega_0+\theta_0)}+\frac{D_0(p)}{\omega_0+\theta_0}\left(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)+1}{(\omega_0+\theta_0)t^2}\right)\left(1+\frac{1}{\omega_0+\theta_0}\right)\right.$$
$$\left.-\frac{D_0(p)}{N(\omega_0+\theta_0)}e^{(\omega_0+\theta_0)t}\right)+H_1\left(-\frac{D_0(p)}{2(\omega_0+\theta_0)}+\frac{2D_0(p)t}{3N(\omega_0+\theta_0)}\right.$$
$$-\frac{D_0(p)}{2n(\omega_0+\theta_0)^2}+\frac{D_0(p)}{(\omega_0+\theta_0)}\left(\frac{\omega_0te^{(\omega_0+\theta_0)t}+\theta_0te^{(\omega_0+\theta_0)t}+1}{(\omega_0+\theta_0)^2t^2}\right)\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)$$
$$\left.-\frac{D_0(p)}{N(\omega_0+\theta_0)}\left(\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\right)\right)+H_2\left(-\frac{2D_0(p)t}{3(\omega_0+\theta_0)}+\frac{3D_0(p)t^2}{4N(\omega_0+\theta_0)}-\frac{2D_0(p)t}{3N(\omega_0+\theta_0)^2}\right.$$
$$+\frac{D_0(p)}{(\omega_0+\theta_0)}\left(\frac{-\theta_0^2t^2+2\omega_0\theta_0t^2-2\theta_0te^{(\omega_0+\theta_0)t}-\omega_0te^{(\omega_0+\theta_0)t}+2e^{(\omega_0+\theta_0)t}+\omega_0^2t^2-2}{(\omega_0+\theta_0)^3t^2}\right)$$
$$\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)-\frac{D_0(p)}{N(\omega_0+\theta_0)}\left(\frac{2e^{(\omega_0+\theta_0)t}-2t(\omega_0+\theta_0)-2}{(\omega_0+\theta_0)^2}\right)\right)$$
$$-\left[P\left(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{D_0(p)}{\omega_0+\theta_0}\left(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)}{t^2}\right)\right.\right.$$
$$\left.\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)-\frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\right)\right]$$
$$-\left[d\left(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{D_0(p)}{\omega_0+\theta_0}\left(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)+1}{t^2}\right)\right.\right.$$
$$\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)-\frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\right)+\frac{D_0(p)}{2N}\left(1-\frac{\omega_0}{\omega_0+\theta_0}\right.$$
$$\left.\left.-\frac{\omega_0D_0(p)}{\omega_0+\theta_0}\left(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)+1}{(\omega_0+\theta_0)t^2}\right)\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{\omega_0D_0(p)}{(\omega_0+\theta_0)n}e^{(\omega_0+\theta_0)t}\right)\right]$$
$$+\frac{N_1v_ce^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t)-1}{\omega_0+\theta_0}+t_e(1-\lambda_0(1-e^{\mu_0\epsilon}))+\frac{d_tv_c}{\omega_0+\theta_0}\left[\left(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0)-1\right)\right.$$
$$\left.\left(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)\right)\left(\frac{1-e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}}{k}\right)\right]=0.$$

(31)

The first partial derivative of $TC(p,t,\epsilon)$ with respect to ϵ is

$$\frac{\partial TC(p,t,\epsilon)}{\partial \epsilon} = t_e\lambda_0e^{-\mu_0\epsilon}\ln(\epsilon)\left[\left(2\tau+\frac{d_tv_c}{\omega_0+\theta_0}\left(e^{(\omega_0+\theta_0)t}-1\right)\frac{c_ev_c}{\omega_0+\theta_0}\right)\right.$$
$$\left(e^{(\omega_0+\theta_0)t}\left(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\right)\right. \tag{32}$$
$$\left.+\left(c_dv_c\right)\left(e^{(\omega_0+\theta_0)t}\left(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\right)\right)\right].$$

6.3.1 Example 3

Let's look at the identical data from Example 1. Assume for the moment that the price-dependent component of demand for the perishable commodity has the following the price exponential demand, function: $D_0(p) = 2000e^{-0.2p}$. By using the algorithm proposed, the following optimal solution for the inventory model is determined: $p^* = 9.5058$ dollars per unit, $t^* = 0.6087$ weeks, $\epsilon^* = 0.2590$, $q^* = 120.36$ units, and $TC^*(p^*,T^*,\epsilon^*) = 563.3379$ euros per week. The total cost function is strictly convex and has a positive definite Hessian matrix; the Hessian determinant is less than zero. Thus, the solution is the optimal. It is ensured that the solution corresponds a global minimum (see Figure 4).

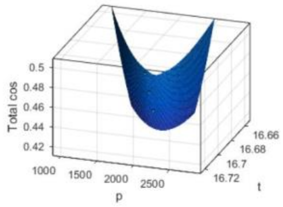

Fig. 4 *Convex property of the total cost TC(p,t,ϵ) when the pricedependent demand function is exponential..*

6.4 Optimal inventory policy using the price-dependent logit demand function with carbon emission

$$D^0(p) = \frac{0}{(1+e^{b_0 p})} \quad a$$

The first partial derivative of $TC(p,t,\epsilon)$ with respect to p is

$$\frac{\partial TC(p,t,\epsilon)}{\partial p} = \left(-H\frac{a_0 b_0 e^{b_0 p}}{(1+e^{b_0 p})^2}\right)\left[\frac{1}{\omega_0+\theta_0}\left[-1+\frac{t}{2N}-\frac{1}{N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\right.$$
$$\left.\left[\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\right]\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]+\left(-H_1\frac{a_0 b_0 e^{b_0 p}}{(1+e^{b_0 p})^2}\right)\left[\frac{t}{\omega_0+\theta_0}\right.$$
$$\left.\left[-\frac{1}{2}+\frac{t}{3N}-\frac{1}{2N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{e^{(\omega_0+\theta_0)t}-1}{(\omega_0+\theta_0)}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right]$$
$$+\left(H_2\frac{a_0 b_0 e^{b_0 p}}{(1+e^{b_0 p})^2}\right)\left[\frac{t^2}{\omega_0+\theta_0}\left[-\frac{1}{3}+\frac{t}{4N}-\frac{1}{3N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\right.$$
$$\left.\left[\frac{2e^{(\omega_0+\theta_0)t}-[t(\omega_0+\theta_0)(t(\omega_0+\theta_0)+2)]-2}{(\omega_0+\theta_0)^3}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right]$$
$$+\left(-\frac{a_0 b_0 e^{b_0 p}}{(1+e^{b_0 p})^2}\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$-\left(-d\frac{a_0 b_0 e^{b_0 p}}{(1+e^{b_0 p})^2}\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(-1-\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\right.$$
$$\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]-1+\frac{t}{2N}+\frac{\omega_0}{(\omega_0+\theta_0)}\left(1-\frac{t}{2N}+\frac{1}{N(\omega_0+\theta_0)}\right)$$
$$+\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\left(\frac{\omega_0}{(\omega_0+\theta_0)t}\right)\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\bigg]\bigg]=0.$$

(33)

$$
\begin{aligned}
&\frac{\partial TC(p,t,\epsilon)}{\partial t} = -\Big[H\Big(\frac{D_0(p)}{2N(\omega_0+\theta_0)} + \frac{D_0(p)}{\omega_0+\theta_0}\Big(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)+1}{(\omega_0+\theta_0)t^2}\Big)\Big(1+\frac{1}{\omega_0+\theta_0}\Big)\\
&- \frac{D_0(p)}{N(\omega_0+\theta_0)}e^{(\omega_0+\theta_0)t}\Big) + H_1\Big(-\frac{D_0(p)}{2(\omega_0+\theta_0)} + \frac{2D_0(p)t}{3N(\omega_0+\theta_0)}\\
&- \frac{D_0(p)}{2n(\omega_0+\theta_0)^2} + \frac{D_0(p)}{(\omega_0+\theta_0)}\Big(\frac{\omega_0 t e^{(\omega_0+\theta_0)t} + \theta_0 t e^{(\omega_0+\theta_0)t}+1}{(\omega_0+\theta_0)^2 t^2}\Big)\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big)\\
&- \frac{D_0(p)}{N(\omega_0+\theta_0)}\Big(\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Big)\Big) + H_2\Big(-\frac{2D_0(p)t}{3(\omega_0+\theta_0)} + \frac{3D_0(p)t^2}{4N(\omega_0+\theta_0)} - \frac{2D_0(p)t}{3N(\omega_0+\theta_0)^2}\\
&+ \frac{D_0(p)}{(\omega_0+\theta_0)}\Big(\frac{-\theta_0^2t^2+2\omega_0\theta_0t^2-2\theta_0te^{(\omega_0+\theta_0)t}-\omega_0te^{(\omega_0+\theta_0)t}+2e^{(\omega_0+\theta_0)t}+\omega_0^2t^2-2}{(\omega_0+\theta_0)^3t^2}\Big)\\
&\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) - \frac{D_0(p)}{N(\omega_0+\theta_0)}\Big(\frac{2e^{(\omega_0+\theta_0)t}-2t(\omega_0+\theta_0)-2}{(\omega_0+\theta_0)^2}\Big)\Big)\\
&- \Big[P\Big(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) + \frac{D_0(p)}{\omega_0+\theta_0}\Big(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)}{t^2}\Big)\\
&\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) - \frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Big)\Big]\\
&- \Big[d\Big(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big)+\frac{D_0(p)}{\omega_0+\theta_0}\Big(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)+1}{t^2}\Big)\\
&\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) - \frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Big) + \frac{D_0(p)}{2N}\Big(1-\frac{\omega_0}{\omega_0+\theta_0}\\
&- \frac{\omega_0 D_0(p)}{\omega_0+\theta_0}\Big(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)+1}{(\omega_0+\theta_0)t^2}\Big)\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) + \frac{\omega_0D_0(p)}{(\omega_0+\theta_0)n}e^{(\omega_0+\theta_0)t}\Big)\Big]\\
&+ \frac{N_1v_ce^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t)-1}{\omega_0+\theta_0} + t_e(1-\lambda_0(1-e^{\mu_0\epsilon})) + \frac{d_tv_c}{\omega_0+\theta_0}\Big[\Big(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0)-1\Big)\\
&\Big(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)\Big)\Big(\frac{1-e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}}{k}\Big)\Big] = 0.
\end{aligned}
$$

(34)

The first partial derivative of $TC(p,t,\epsilon)$ with respect to ϵ is

$$
\begin{aligned}
&\frac{\partial TC(p,t,\epsilon)}{\partial\epsilon} = t_e\lambda_0e^{-\mu_0\epsilon}\ln(\epsilon)\Big[\Big(2\tau+\frac{d_tv_c}{\omega_0+\theta_0}\Big(e^{(\omega_0+\theta_0)t}-1\Big)\frac{c_ev_c}{\omega_0+\theta_0}\Big)\\
&\Big(e^{(\omega_0+\theta_0)t}\Big(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Big)\\
&+\Big(c_dv_c\Big)\Big(e^{(\omega_0+\theta_0)t}\Big(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}-1}{k}\Big)\Big)\Big].
\end{aligned} \tag{35}
$$

6.4.1 Example 4

Let's look at the identical data from Example 1. Assume for the moment that the price-dependent term of the demand for the fresh article following a logit demand function: $D_0(p) = (9000/(1+e^{0.3p}))$. By using the algorithm proposed, the following optimal solution for the inventory model is determined: $p^* = 7.9635$ dollars per unit, $t^* = 0.4582$ weeks, $\epsilon^* = 0.2378$, $q^* = 230.1214$ units, and $TC^*(p^*,T^*,\epsilon^*) = 1203.467$ dollars per week. The total cost function is strictly convex and has a positive definite Hessian matrix; the Hessian determinant is less than zero. Thus, the solution is the optimal. It is ensured that the solution corresponds a global minimum (see Figure 5).

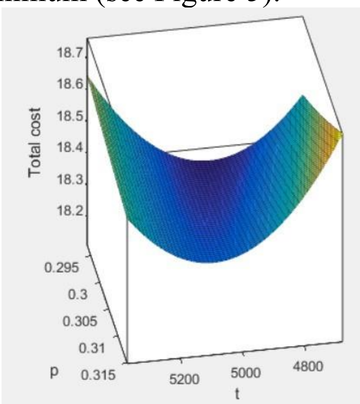

Fig. 5 *Convex property of the total cost $TC(p,t,\epsilon)$ when the pricedependent demand function is logit.*

6.5 Optimal inventory policy using the price-dependent logarithmic demand function with carbon emission $D_0(p) = (a_0 - b_0)\ln(p)$

The first partial derivative of $TC(p,t,\epsilon)$ with respect to p is

$$\frac{\partial TC(p,t,\epsilon)}{\partial p} = \left(-H\frac{-b_0}{p}\right)\left[\frac{1}{\omega_0+\theta_0}\left[-1+\frac{t}{2N}-\frac{1}{N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\right.$$
$$\left.\left[\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\right]\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]+\left(-H_1\frac{-b_0}{p}\right)\left[\frac{t}{\omega_0+\theta_0}\right.$$
$$\left.\left[-\frac{1}{2}+\frac{t}{3N}-\frac{1}{2N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{e^{(\omega_0+\theta_0)t}-1}{(\omega_0+\theta_0)}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right]$$
$$+\left(H_2\frac{-b_0}{p}\right)\left[\frac{t^2}{\omega_0+\theta_0}\left[-\frac{1}{3}+\frac{t}{4N}-\frac{1}{3N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\right.$$
$$\left.\left[\frac{2e^{(\omega_0+\theta_0)t}-[t(\omega_0+\theta_0)(t(\omega_0+\theta_0)+2)]-2}{(\omega_0+\theta_0)^3}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\right.$$
$$+\left(\frac{b_0}{p}\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$-\left(d\frac{b_0}{p}\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(-1-\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\right.$$
$$\left.\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]-1+\frac{t}{2N}+\frac{\omega_0}{(\omega_0+\theta_0)}\left(1-\frac{t}{2N}+\frac{1}{N(\omega_0+\theta_0)}\right)$$
$$\left.+\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\left(\frac{\omega_0}{(\omega_0+\theta_0)t}\right)\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\Bigg]=0.$$

(36)

The first partial derivative of $TC(p,t,\epsilon)$ with respect to t is

$$\frac{\partial TC(p,t,\epsilon)}{\partial t} = -\Bigg[H\Bigg(\frac{D_0(p)}{2N(\omega_0+\theta_0)}+\frac{D_0(p)}{\omega_0+\theta_0}\left(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)+1}{(\omega_0+\theta_0)t^2}\right)\left(1+\frac{1}{\omega_0+\theta_0}\right)$$
$$-\frac{D_0(p)}{N(\omega_0+\theta_0)}e^{(\omega_0+\theta_0)t}\Bigg)+H_1\Bigg(-\frac{D_0(p)}{2(\omega_0+\theta_0)}+\frac{2D_0(p)t}{3N(\omega_0+\theta_0)}$$
$$-\frac{D_0(p)}{2n(\omega_0+\theta_0)^2}+\frac{D_0(p)}{(\omega_0+\theta_0)}\left(\frac{\omega_0 te^{(\omega_0+\theta_0)t}+\theta_0 te^{(\omega_0+\theta_0)t}+1}{(\omega_0+\theta_0)^2t^2}\right)\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)$$
$$-\frac{D_0(p)}{N(\omega_0+\theta_0)}\left(\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\right)\Bigg)+H_2\Bigg(-\frac{2D_0(p)t}{3(\omega_0+\theta_0)}+\frac{3D_0(p)t^2}{4N(\omega_0+\theta_0)}-\frac{2D_0(p)t}{3N(\omega_0+\theta_0)^2}$$
$$+\frac{D_0(p)}{(\omega_0+\theta_0)}\left(\frac{-\theta_0^2t^2+2\omega_0\theta_0t^2-2\theta_0te^{(\omega_0+\theta_0)t}-\omega_0te^{(\omega_0+\theta_0)t}+2e^{(\omega_0+\theta_0)t}+\omega_0^2t^2-2}{(\omega_0+\theta_0)^3t^2}\right)$$
$$\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)-\frac{D_0(p)}{N(\omega_0+\theta_0)}\left(\frac{2e^{(\omega_0+\theta_0)t}-2t(\omega_0+\theta_0)-2}{(\omega_0+\theta_0)^2}\right)\Bigg)$$
$$-\Bigg[P\Bigg(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{D_0(p)}{\omega_0+\theta_0}\left(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)}{t^2}\right)$$
$$\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)-\frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Bigg)\Bigg]$$
$$-\Bigg[d\Bigg(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{D_0(p)}{\omega_0+\theta_0}\left(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)+1}{t^2}\right)$$
$$\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)-\frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Bigg)+\frac{D_0(p)}{2N}\left(1-\frac{\omega_0}{\omega_0+\theta_0}\right.$$
$$\left.-\frac{\omega_0D_0(p)}{\omega_0+\theta_0}\left(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)+1}{(\omega_0+\theta_0)t^2}\right)\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{\omega_0D_0(p)}{(\omega_0+\theta_0)n}e^{(\omega_0+\theta_0)t}\right)\Bigg]$$
$$+\frac{N_1v_ce^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t)-1}{\omega_0+\theta_0}+t_e(1-\lambda_0(1-e^{\mu_0\epsilon}))+\frac{d_tv_c}{\omega_0+\theta_0}\Bigg[\left(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0)-1\right)$$
$$\left(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)\right)\left(\frac{1-e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)}{k+(\omega_0+\theta_0)}+\frac{e^{-kt}}{k}\right)\Bigg]=0.$$

(37)

The first partial derivative of $TC(p,t,\epsilon)$ with respect to ϵ is

$$\frac{\partial TC(p,t,\epsilon)}{\partial \epsilon} = t_e\lambda_0 e^{-\mu_0\epsilon}\ln(\epsilon)\Bigg[\left(2\tau + \frac{d_t v_c}{\omega_0+\theta_0}\left(e^{(\omega_0+\theta_0)t}-1\right)\frac{c_e v_c}{\omega_0+\theta_0}\right)$$
$$\left(e^{(\omega_0+\theta_0)t}\left(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}-1}{k}\right)\right.$$
$$+\left(c_d v_c\right)\left(e^{(\omega_0+\theta_0)t}\left(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}-1}{k}\right)\right)\Bigg]. \quad (38)$$

6.5.1 Example 5

Let's look at the identical data from Example 1. Assume for the moment that pricedependent factor of the demand for the fresh produce follows a logarithmic demand function: $D_0(p) = 95-21\ln p$. By using the algorithm proposed, the following optimal solution for the inventory model is determined: $p^* = 38.1535$ dollars per unit, $t^* = 0.8629$ weeks,$\epsilon^* = 9.6536$, $q^* = 0.22090$ units, and $TC^*(p^*,T^*,\epsilon^*) = 115.4864$ dollar per week. The total cost function is strictly convex and has a positive definite Hessian matrix; the Hessian determinant is less than zero. Thus, the solution is the optimal. It is ensured that the solution corresponds a global minimum (see Figure 6).

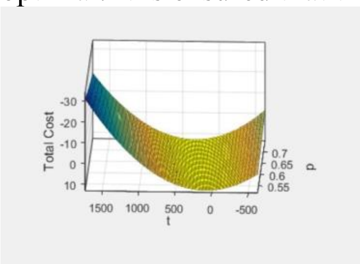

Fig. 6 *Convex property of the total cost $TC(p,t,\epsilon)$ when the pricedependent demand function is logarithmic.*

6.6 Optimal inventory policy using the price-dependent polynomial demand function with carbon emission $D_0(p) = (a_0 - b_0)p^m$.

The first partial derivative of $TC(p,t,\epsilon)$ with respect to p is

$$\frac{\partial TC(p,t,\epsilon)}{\partial p} = \left(-H(-mb_0p^{m-1})\right)\left[\frac{1}{\omega_0+\theta_0}\left[-1+\frac{t}{2N}-\frac{1}{N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\right.$$
$$\left[\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\right]\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\Bigg]+\left(-H_1(-mb_0p^{m-1})\right)\left[\frac{t}{\omega_0+\theta_0}\right.$$
$$\left[-\frac{1}{2}+\frac{t}{3N}-\frac{1}{2N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\left[\frac{e^{(\omega_0+\theta_0)t}-1}{(\omega_0+\theta_0)}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]\Bigg]$$
$$+\left(H_2(-mb_0p^{m-1})\right)\left[\frac{t^2}{\omega_0+\theta_0}\left[-\frac{1}{3}+\frac{t}{4N}-\frac{1}{3N(\omega_0+\theta_0)}\right]+\frac{1}{t(\omega_0+\theta_0)}\right.$$
$$\left[\frac{2e^{(\omega_0+\theta_0)t}-[t(\omega_0+\theta_0)(t(\omega_0+\theta_0)+2)]-2}{(\omega_0+\theta_0)^3}\right]\left[\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$+\left((-mb_0p^{m-1})\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\right]$$
$$-\left(d(-mb_0p^{m-1})\right)\left[\frac{1}{(\omega_0+\theta_0)t}\left(-1-\frac{1}{N(\omega_0+\theta_0)}\right)+\frac{e^{(\omega_0+\theta_0)t}}{(\omega_0+\theta_0)t}\right.$$
$$\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\Bigg]-1+\frac{t}{2N}+\frac{\omega_0}{(\omega_0+\theta_0)}\left(1-\frac{t}{2N}+\frac{1}{N(\omega_0+\theta_0)}\right)$$
$$+\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\left(\frac{\omega_0}{(\omega_0+\theta_0)t}\right)\left(1+\frac{1}{N(\omega_0+\theta_0)}-\frac{t}{N}\right)\Bigg]\Bigg]=0.$$

(39)

The first partial derivative of $TC(p,t,\epsilon)$ with respect to t is

$$\frac{\partial TC(p,t,\epsilon)}{\partial t} = -\Big[H\Big(\frac{D_0(p)}{2N(\omega_0+\theta_0)} + \frac{D_0(p)}{\omega_0+\theta_0}\Big(\frac{e^{(\omega_0+\theta_0)t}(\omega_0 t+\theta_0 t-1)+1}{(\omega_0+\theta_0)t^2}\Big)\Big(1+\frac{1}{\omega_0+\theta_0}\Big)$$
$$-\frac{D_0(p)}{N(\omega_0+\theta_0)}e^{(\omega_0+\theta_0)t}\Big) + H_1\Big(-\frac{D_0(p)}{2(\omega_0+\theta_0)} + \frac{2D_0(p)t}{3N(\omega_0+\theta_0)}$$
$$-\frac{D_0(p)}{2n(\omega_0+\theta_0)^2} + \frac{D_0(p)}{(\omega_0+\theta_0)}\Big(\frac{\omega_0 t e^{(\omega_0+\theta_0)t}+\theta_0 t e^{(\omega_0+\theta_0)t}+1}{(\omega_0+\theta_0)^2t^2}\Big)\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big)$$
$$-\frac{D_0(p)}{N(\omega_0+\theta_0)}\Big(\frac{e^{(\omega_0+\theta_0)t}-1}{\omega_0+\theta_0}\Big)\Big) + H_2\Big(-\frac{2D_0(p)t}{3(\omega_0+\theta_0)} + \frac{3D_0(p)t^2}{4N(\omega_0+\theta_0)} - \frac{2D_0(p)t}{3N(\omega_0+\theta_0)^2}$$
$$+\frac{D_0(p)}{(\omega_0+\theta_0)}\Big(\frac{-\theta_0^2t^2+2\omega_0\theta_0t^2-2\theta_0te^{(\omega_0+\theta_0)t}-\omega_0te^{(\omega_0+\theta_0)t}+2e^{(\omega_0+\theta_0)t}+\omega_0^2t^2-2}{(\omega_0+\theta_0)^3t^2}\Big)$$
$$\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) - \frac{D_0(p)}{N(\omega_0+\theta_0)}\Big(\frac{2e^{(\omega_0+\theta_0)t}-2t(\omega_0+\theta_0)-2}{(\omega_0+\theta_0)^2}\Big)\Big)$$
$$-\Big[P\Big(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) + \frac{D_0(p)}{\omega_0+\theta_0}\Big(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)}{t^2}\Big)$$
$$\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) - \frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Big)\Big]$$
$$-\Big[d\Big(\frac{D_0(p)}{(\omega_0+\theta_0)t^2}\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) + \frac{D_0(p)}{\omega_0+\theta_0}\Big(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)+1}{t^2}\Big)$$
$$\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) - \frac{D_0(p)}{N(\omega_0+\theta_0)}(\omega_0+\theta_0)e^{(\omega_0+\theta_0)t}\Big) + \frac{D_0(p)}{2N}\Big(1-\frac{\omega_0}{\omega_0+\theta_0}$$
$$-\frac{\omega_0D_0(p)}{\omega_0+\theta_0}\Big(\frac{e^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t-1)+1}{(\omega_0+\theta_0)t^2}\Big)\Big(1+\frac{1}{N(\omega_0+\theta_0)}\Big) + \frac{\omega_0D_0(p)}{(\omega_0+\theta_0)n}e^{(\omega_0+\theta_0)t}\Big)\Big]$$
$$+\frac{N_1v_ce^{(\omega_0+\theta_0)t}(\omega_0t+\theta_0t)-1}{\omega_0+\theta_0} + t_e(1-\lambda_0(1-e^{\mu_0\epsilon})) + \frac{d_tv_c}{\omega_0+\theta_0}\Big[\Big(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0)-1\Big)$$
$$\Big(e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)\Big)\Big(\frac{1-e^{(\omega_0+\theta_0)t}(\omega_0+\theta_0-1)}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}}{k}\Big)\Big] = 0.$$
(40)

The first partial derivative of $TC(p,t,\epsilon)$ with respect to ϵ is

$$\frac{\partial TC(p,t,\epsilon)}{\partial \epsilon} = t_e\lambda_0e^{-\mu_0\epsilon}\ln(\epsilon)\Big[\Big(2\tau + \frac{d_tv_c}{\omega_0+\theta_0}\Big(e^{(\omega_0+\theta_0)t}-1\Big)\frac{c_ev_c}{\omega_0+\theta_0}\Big)$$
$$\Big(e^{(\omega_0+\theta_0)t}\Big(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}-1}{k}\Big)$$
$$+\Big(c_dv_c\Big)\Big(e^{(\omega_0+\theta_0)t}\Big(\frac{(1-e^{-(k+(\omega_0+\theta_0)t)})}{k+(\omega_0+\theta_0)} + \frac{e^{-kt}-1}{k}\Big)\Big]. \quad (41)$$

6.6.1 Example 6

Let's look at the identical data from Example 1. Assume for the moment that the price-dependent portion of the demand for the fresh produce following a polynomial demand function: $D_0(p) = 4000 - 2p^3$. By using the algorithm proposed, the following optimal solution for the inventory model is determined: $p^* = 8.4752$ dollar per unit, $t^* = 0.2156$ weeks, $\epsilon^* = 0.2345$, $q^* = 487.724$ units, and $TC^*(p^*,T^*,\epsilon^*) = 8082.7$ dollars per week. The total cost function is strictly convex and has a positive definite Hessian matrix; the Hessian determinant is less than zero. Thus, the solution is the optimal. It is ensured that the solution corresponds a global minimum (see Figure 7).

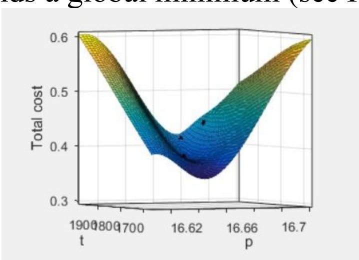

Fig. 7 *Convex property of the total cost $TC(p,t,\epsilon)$ when the pricedependent demand function is polynomial.*

7 Sensitivity Analysis

The sensitivity analysis performed to investigate the impact of the inventory model's input parameters on the choice variables (p,t,q,ϵ) and total cost $TC(p,T,\epsilon)$ is shown in this section. For each of the six price-dependent demands, a sensitivity analysis is conducted. Each input parameter has a unique value, whereas the other input data are fixed. Tables 1 - 6 display the findings. Additionally included below are some observations and managerial insights. Tables 1 - 6 show how the input parameters affect the price-dependent linear, isoelastic, exponential, logit, logarithmic, and polynomial demand functions' optimal solutions for p,t,q,ϵ and the total cost, respectively.

The numerical experimentation displayed in Tables 1–5 provides the foundation for Table 6. As the parameters of the inventory model are increased, Table 6 displays the behaviour of the variables as well as the total cost. Tables 1–6 provide the observations and managerial insights that follow.

- In all price-demand functions, the maximum shelf-life (N) of a fresh item and the ordering cost (O) have an identical effect on the inventory policy and overall profit. Longer product shelf-life N leads to higher prices, longer cycle times, and greater order quantities. This implies that a marketer can charge more for a fresh product because its shelf life is greater and it loses its appeal over time. Prices and inventory cycle times rise in tandem with an increase in ordering cost O, while overall profit typically decreases. Then, in order to increase profitability, managers ought to take steps to reduce ordering costs.
- Increased values of the demand sensitivity to stock level (ω_0) parameter lead to higher prices, longer inventory cycle times, and larger profitability. With the exception of the logarithmic function, which shows a tendency for the ideal price and inventory cycle time to decrease as this parameter grows, this behaviour is seen in five price-demand functions. It is discovered that greater values of the demand sensitivity to stock level (ω_0) parameter lead to bigger order quantities for all price-demand relationships. In this instance, taking high values of the ω_0 parameter would be advised in order to produce high values of profits because it does not directly depend on the decision maker. This indicates that managers are more eager to display greater amounts of inventory on their shelves.
- Increased values of the demand sensitivity to stock level (ω_0) parameter lead to higher prices, longer inventory cycle times, and larger profitability. With the exception of the logarithmic function, which shows a tendency for the ideal price and inventory cycle time to decrease as this parameter grows, this behaviour is seen in five price-demand functions. Higher values of the parameter of demand sensitivity to the stock level (ω_0) are found to result in a bigger order quantity for all price-demand functions. In this instance, it is suggested to use high values of the ω_0 parameter to produce high values of profits because it does not directly depend on the decision maker. This indicates that managers are more eager to display greater amounts of inventory on their shelves.
- The price and inventory cycle time increase marginally as the purchase cost (P) rises. On the other hand, as this expense rises, the profit decreases. Every price-demand function exhibits this behaviour. Thus, the recommendation would be to create low-cost purchasing procedures in order to increase profits independent of how demand functions and the price that is applied. Finding different suppliers who can provide goods at a lesser price without sacrificing quality is one way.
- The holding cost function (H,H_1,H_2) and the deterioration cost (d) have larger coefficients, which lead to a marginally higher selling price but a smaller cost. This pattern of behaviour is seen in all price-demand functions. Conversely, for the rest of the price-demand relationships, the inventory cycle time drops and grows for the logarithmic function. Decision-makers are typically urged to put strategies in place to have minimal product deterioration costs in order to boost total cost. Lower values for the holding cost function's component parts also need to be taken into account.
- In the logarithmic, polynomial, and linear functions, higher prices and shorter inventory cycle times are attained by fixing the parameter of the demand's sensitivity to price (b_0) and raising the values of the demand's scale parameter (a_0). Conversely, as the scale parameter rises in the isoelastic, exponential, and logit price demand functions, lower prices and lower inventory cycles are seen. Greater advantages are obtained in all price-demand functions when the value of an is raised while the value of b_0 remains constant. Therefore, it is suggested that greater values be maintained for this parameter.
- In all price-demand functions, larger values of the demand sensitivity to price (b_0) lead to lower selling prices and profits, but longer inventory cycles when the scale parameter of the demand (a_0) is kept constant. Therefore, regardless of the price demand utilized, low values of this parameter b_0 should be used to ensure that overall profits are always higher.

8 Conclusion

An inventory model incorporating price, stock, and time dependent demand is developed in this research effort. Zero-ending inventory is assumed, and both the physical deterioration and the state of freshness degradation over time are taken into account. Price-dependent demand functions of the following six varieties are examined: polynomial, logit, exponential, isoelastic, and logit. When dealing with perishable goods, the full cycle takes into account both a salvaged value and a deteriorating cost. There is also a nonlinear, time-dependent holding cost—more precisely, a quadratic-type function. When dealing with perishable goods, the full cycle takes into account both a salvaged value and a deteriorating cost. There is also a nonlinear, time-dependent holding cost more specifically, a quadratic-type function.

The inventory model uses an algorithm to calculate the best values for the order amount, price, and inventory cycle time. There are certain numerical examples given, and a sensitivity analysis for each and every input parameter is shown. A comparable pattern in the selling price, inventory cycle time, quantity to order, and total cost is produced by an increase in the ordering cost (O), purchasing cost (P), and shelf-life (N). This was discovered by analysing the behaviour of the choice variables and total cost. Furthermore, an increase in the value of the shelf-life (N) results in an increment in price, inventory cycle time, quantity ordered, and costs generated for all functions. Finally, by raising the purchasing cost for all functions, there is an increase in both the price and the inventory cycle time; however, the quantity to order and total cost tend to increase. As the ordering cost increases (O), price, the inventory cycle time, and quantity ordered also increase for all functions. Nevertheless, the costs show an increasing trend.

This study expands and significantly advances the state-of-the-art in the field of inventory, concentrating on perishable goods whose demand is reliant on price, stock, and time. The inventory model under study has certain shortcomings, which reveal various avenues for further investigation and extension. It is first necessary to investigate a model that permits shortages with either complete or partial backlog. Secondly, research should be done on the advantages and trade-offs of investing in preservation technologies. Third, the suggested inventory model might incorporate the freshness deterioration of the non-instantaneous item. Lastly, additional elements can also be researched, such as implementing discount programmers or advertising campaigns.

Competing interests

The authors declare that they have no competing interests.

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Author's contributions

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Parameter		p	T	ϵ	Q	TC
	0.5	16.62065	0.2795503	0.2431	53.61395	1456.227
	0.6	16.63586	0.3111456	0.23923	61.92978	1626.649
	0.7	16.65028	0.3415362	0.24316	69.96242	1761.836
	0.8	16.66419	0.3711902	0.24390	77.8329	1873.133
	0.9	16.6778	0.4004517	0.23450	85.63123	1967.361
N	1	16.69124	0.4295922	0.23682	93.42941	2048.903
	1.1	16.70463	0.4588389	0.2431	101.2887	2120.728
	1.2	16.71807	0.4883923	0.23626	109.2643	2184.928
	1.3	16.73165	0.5184379	0.24409	117.4082	2243.032

1.4	16.74546	0.5491541	0.23822	125.7715	2296.187
1.5	16.75956	0.5807188	0.22090	134.4061	2345.277
250	16.69124	0.4295922	0.23451	93.42941	2048.903
300	16.70497	0.4666712	0.24390	99.6798	1939.786
O 350	16.71696	0.5001394	0.2431	105.0448	1838.465
400	16.72758	0.5307686	0.22837	109.7159	1743.321
450	16.73707	0.5590932	0.24316	113.8253	1653.228
500	16.74562	0.5755007	0.23142	117.4688	1567.368
0.25	16.68249	0.3995219	0.23626	83.91483	1919.088
0.35	16.6858	0.4107697	0.26353	87.4505	1969.198
$\omega 0$ 0.45	16.68936	0.4230415	0.23626	91.33854	2021.689
0.5	16.69124	0.4295922	0.24316	93.42941	2048.903
0.55	16.69317	0.4364355	0.23450	95.6265	2076.807
0.03	16.68352	0.4299946	0.24390	93.1671	2051.515
0.04	16.68737	0.4297943	0.25901	93.2983	2050.211
$\theta 0$ 0.05	16.69124	0.4295922	0.2431	93.4294	2048.903
0.06	16.6951	0.4293881	0.24873	93.5602	2046.593
0.07	16.69898	0.4291822	0.24390	93.69074	2046.279
4	16.18122	0.4156198	0.23599	94.76367	2268.809
4.5	16.43615	0.4224792	0.24390	94.1051	2157.583
P 5	16.69124	0.4295922	0.23451	93.4294	2048.903
5.5	16.94647	0.4369741	0.23626	92.73557	1942.772
6	17.20186	0.4446421	0.21824	92.02275	1839.193
1.65,0.05,0.15	16.68038	0.420920	0.23789	93.74612	2053.729
1.70,0.10,0.20	16.68582	0.420255	0.23626	93.58753	2051.314
H 1.75, 0.15, 0.25	16.69124	0.429592	0.2431	93.42941	2048.903
1.80, 0.20, 0.30	16.69664	0.428930	0.24317	93.27176	2046.496
1.85, 0.25, 0.35	16.70203	0.428271	0.23142	93.11458	2044.093
1	16.68655	0.420052	0.24390	92.54633	2050.972
1.5	16.68889	0.429822	0.23822	92.48785	2049.937
d 2	16.69124	0.429592	0.24390	92.42941	2048.903
2.5	16.69358	0.429362	0.23822	92.37101	2047.87
3	16.69592	0.429132	0.22837	92.31266	2046.837
0.6	16.69498	0.429224	0.23593	93.33599	2047.25
0.7	16.69311	0.429408	0.24390	93.38269	2048.076
$b0$ 0.8	16.69124	0.429592	0.23682	93.42941	2048.903
0.9	16.68936	0.429776	0.24316	93.47616	2049.731
1	16.68748	0.429960	0.22090	93.52293	2050.558
360	10.79361	0.760094	0.23793	68.96747	230.0563
480	13.73238	0.544978	0.2431	84.21757	955.0452
$a0,b0$ fixed 600	16.69124	0.429592	0.23450	93.42941	2048.903
720	19.66264	0.355883	0.24390	99.8173	3507.115
840	22.64165	0.304190	0.23626	104.5754	5327.862
5	61.60045	0.207419	0.21824	57.93376	14328.26
10	31.63771	0.294692	0.2431	75.55008	6004.921
$b0,a0$ fixed 20	16.69124	0.429592	0.24390	93.42941	2048.903
25	13.71425	0.492487	0.24316	98.05132	1312.228
30	11.73633	0.556856	0.2431	100.7333	846.1779

Table 1 Sensitivity analysis of the optimal solution for the price-dependent linear demand function.

Parameter	p	T	ϵ	Q	TC
0.5	17.1207	0.1906021	0.24316	86.68895	4385.067
0.6	17.19569	0.2131854	0.24409	98.39292	4637.565

N	0.7	17.2679	0.2350845	0.22837	109.6674	4838.074
	0.8	17.33853	0.2566374	0.23451	120.6934	5003.362
	0.9	17.4084	0.2781024	0.23787	131.6066	5143.51
	1	17.4785	0.2996932	0.22090	142.5169	5265.004
	1.1	17.5492	0.3215992	0.23450	153.5197	5372.237
	1.2	17.62116	0.3439997	0.23682	164.7028	5468.315
	1.3	17.6949	0.3670726	0.23593	176.1515	5555.509
	1.4	17.77111	0.3910032	0.24164	187.9524	5635.533
	1.5	17.8502	0.4159907	0.23923	200.197	5709.711
O	250	17.4785	0.2896932	0.23822	142.5169	5265.004
	300	17.5564	0.3168009	0.24873	152.6876	5110.347
	350	17.6259	0.3413822	0.23599	161.5787	4967.135
	400	17.6886	0.3639697	0.24316	169.47	4832.983
	450	17.7459	0.3849329	0.23793	176.5547	4706.234
	500	17.7987	0.4045407	0.21824	182.9726	4585.683
$\omega 0$	0.25	17.4029	0.2705749	0.23789	126.6708	5050.189
	0.35	17.4308	0.2812172	0.23142	132.4547	5132.826
	0.45	17.46176	0.293162	0.25901	138.956	5219.756
	0.5	17.4785	0.2996932	0.20389	142.5169	5265.004
	0.55	17.4961	0.3066393	0.23609	146.3102	5311.546
$\theta 0$	0.03	17.4399	0.3001287	0.25763	142.6865	52760.35
	0.04	17.4592	0.299911	0.24004	142.6018	5267.177
	0.05	17.4785	0.2996932	0.22090	142.5169	5265.004
	0.06	17.4977	0.2994752	0.23787	142.4319	5262.831
	0.07	17.51709	0.2992571	0.24004	142.3467	5260.658
P	4	13.9077	0.2835993	0.23450	183.7868	5804.186
	4.5	15.6942	0.2921076	0.27431	160.8104	5514.016
	5	17.4785	0.2996932	0.25763	142.5169	5265.004
	5.5	19.26106	0.3065394	0.24164	127.6391	5048.084
	6	21.04224	0.3127805	0.24240	115.3246	4856.783
H 2	1.65, 0.05, 0.15	17.42584	0.3107838	0.23451	141.5444	5272.343
	1.70, 0.10, 0.20	17.45222	0.3102363	0.23618	141.0284	5268.667
	1.75, 0.15, 0.25	17.4785	0.3096932	0.24164	141.5169	5265.004
	1.80, 0.20, 0.30	17.5047	0.3091542	0.22837	141.0099	5261.354
	1.85, 0.25, 0.35	17.5308	0.3086195	0.26353	141.5073	5257.717
d	1	17.45472	0.3001169	0.23923	142.9515	5268.293
	1.5	17.46662	0.2999047	0.23872	142.7339	5266.647
	2	17.4785	0.2996932	0.23822	142.5169	5265.004
	2.5	17.49037	0.2994822	0.24317	142.3007	5263.363
	3	17.50223	0.2992719	0.20389	142.0853	5261.724
$b0$	0.6	17.49749	0.299356	0.24390	142.1714	5262.379
	0.7	17.488	0.2995244	0.24316	142.3439	5263.691
	0.8	17.4785	0.2996932	0.15278	142.5169	5265.004
	0.9	17.469	0.2998624	0.23682	142.6904	5266.318
	1	17.45948	0.3000319	0.23593	142.8644	5267.634
$a0, b0$ fixed	360	18.01343	0.4979969	0.23599	68.67333	1349.356
	480	17.65804	0.3628992	0.24164	110.0905	3265.695
	600	17.4785	0.2996932	0.23609	142.5169	5265.004
	720	17.36477	0.2609628	0.24409	170.0939	7305.271
	840	17.2842	0.234065	0.23793	194.5106	9375.999
$b0, a0$ fixed	5	30.32621	0.2305638	0.24873	107.4781	10665.25
	10	21.74959	0.2642978	0.24164	130.4156	7418.431
	20	17.4785	0.2996932	0.25901	142.5169	5265.004

25	14.93154	0.3374883	0.23822	147.2634	3775.648
30	13.24802	0.3782774	0.21824	146.9029	2717.869

Table 2 Sensitivity analysis of the optimal solution for the price-dependent isoelastic demand function.

Parameter		p	T	ϵ	Q	TC
N	0.5	9.31047	0.4181761	0.26353	67.1642	204.8662
	0.6	9.35651	0.4606555	0.22090	79.00251	310.0982
	0.7	9.39787	0.500368	0.23450	90.08083	392.5449
	0.8	9.43602	0.5379941	0.23599	100.5864	459.5593
	0.9	9.47181	0.5740098	0.24164	110.6502	515.5487
	1	9.50582	0.6087656	0.25901	120.3688	563.3379
	1.1	9.53846	0.6425299	0.23142	129.8162	604.8324
	1.2	9.57	0.6755155	0.22837	139.0512	641.3708
	1.3	9.6007	0.7078965	0.23787	148.1221	673.9252
	1.4	9.63073	0.7398184	0.24409	157.069	703.2209
	1.5	9.66023	0.7714064	0.23451	165.9269	729.8121
O	250	9.50582	0.6087656	0.24873	120.3688	563.3379
	300	9.53844	0.6540886	0.23789	126.3607	486.3458
	350	9.56532	0.7143232	0.24164	131.0419	414.4977
	400	9.58741	0.7605352	0.23822	133.6953	347.6073
	450	9.60539	0.8034684	0.23593	137.5174	284.4805
	500	9.61971	0.8436671	0.23609	139.6506	224.5008
$\omega 0$	0.25	9.50397	0.5898278	0.15278	110.1458	497.051
	0.35	9.50459	0.5969651	0.22090	114.0211	522.7045
	0.45	9.50537	0.6046843	0.27154	118.1772	549.4925
	0.5	9.50582	0.6087656	0.24316	120.3688	563.3379
	0.55	9.5063	0.612996	0.25763	122.6407	577.4992
$\theta 0$	0.03	9.48514	0.6094522	0.23729	120.2816	566.9372
	0.04	9.49547	0.6091109	0.23450	120.3257	565.1389
	0.05	9.50582	0.6087656	0.23793	120.3688	563.3379
	0.06	9.5162	0.6084164	0.23787	120.4107	561.5343
	0.07	9.5266	0.6080633	0.23815	120.4516	559.7281
P	4	8.458612	0.5436089	0.22347	139.2348	784.8765
	4.5	8.981914	0.5805406	0.23142	129.5493	667.5296
	5	9.50582	0.6087656	0.23787	120.3688	563.3379
	5.5	10.03024	0.6383456	0.23682	111.6667	470.9582
	6	10.55502	0.6693453	0.24213	103.418	389.1792
H	1.65, 0.05, 0.15	9.47553	0.6010162	0.25901	121.4024	569.4779
	1.70, 0.10, 0.20	9.4907	0.6098884	0.24317	120.8838	566.4016
	1.75, 0.15, 0.25	9.50582	0.6087656	0.23142	120.3688	563.3379
	1.80, 0.20, 0.30	9.52089	0.6076479	0.24398	119.8572	560.2867
	1.85, 0.25, 0.35	9.53591	0.6065353	0.20325	119.3493	557.2478
d	1	9.49339	0.6093265	0.24577	120.7445	564.8144
	1.5	9.49961	0.6090458	0.22090	120.5564	563.5753
	2	9.50582	0.6087656	0.24390	120.3688	562.3379
	2.5	9.51203	0.6084858	0.24227	120.1815	561.1023
	3	9.51824	0.6082064	0.26353	119.9946	560.8685
$b0$	0.6	9.51576	0.6083181	0.15278	120.0693	561.3618
	0.7	9.51079	0.6085417	0.23822	120.2189	562.3493
	0.8	9.50582	0.6087656	0.23609	120.3688	563.3379
	0.9	9.50085	0.6089897	0.22837	120.5189	564.3277
	1	9.49588	0.6092141	0.24873	120.6693	565.3185

360	9.61971	0.8436671	0.20389	69.32532	111.7504
480	9.55693	0.6980697	0.23789	96.95558	328.0764
$a0, b0$ fixed 600	9.50582	0.6087656	0.24164	120.3688	563.3379
720	9.46593	0.5467266	0.24409	141.0386	810.742
840	9.43408	0.5003661	0.21824	159.7423	1066.793
5	14.32865	0.3601014	0.23793	142.3381	3164.293
10	11.0851	0.4784015	0.22090	135.2242	1327.054
$b0, a0$ fixed 20	9.50582	0.6087656	0.23682	120.3688	563.3379
25	8.588089	0.7620412	0.23450	101.6994	199.5329
30	7.976881	0.9533883	0.23599	80.83712	18.17279

Table 3 Sensitivity analysis of the optimal solution for the price-dependent exponential demand function.

Parameter		p	T	ϵ	Q	TC
	0.5	*7.83536*	*0.321024*	*0.23793*	*141.0055*	*764.3444*
	0.6	*7.865511*	*0.3425493*	*0.24227*	*161.432*	*895.2742*
	0.7	*7.892812*	*0.3815668*	*0.24317*	*180.2635*	*997.0164*
	0.8	*7.917971*	*0.4086151*	*0.23593*	*196.8412*	*1079.029*
	0.9	*7.941448*	*0.4340691*	*0.23142*	*214.4026*	*1145.972*
N	*1*	*7.96356*	*0.4582024*	*0.23787*	*230.1214*	*1203.467*
	1.1	*7.98454*	*0.4812223*	*0.23599*	*245.1292*	*1253.953*
	1.2	*8.004563*	*0.5032901*	*0.22090*	*259.5286*	*1295.141*
	1.3	*8.023767*	*0.5245347*	*0.23451*	*273.4013*	*1334.271*
	1.4	*8.04226*	*0.5450609*	*0.23450*	*286.8142*	*1369.265*
	1.5	*8.060133*	*0.5649555*	*0.25901*	*299.8225*	*1399.829*
	250	*7.96356*	*0.4582024*	*0.22837*	*230.1214*	*1204.467*
	300	*7.993896*	*0.5032866*	*0.24409*	*245.2305*	*1102.581*
O	*350*	*8.020562*	*0.544725*	*0.21824*	*256.9851*	*1007.949*
	400	*8.044224*	*0.5832662*	*0.22090*	*265.8674*	*921.8403*
	450	*8.065344*	*0.6194328*	*0.24390*	*277.2142*	*840.0543*
	500	*8.084255*	*0.653607*	*0.24873*	*286.2716*	*762.7174*
	0.25	*7.952328*	*0.4336449*	*0.23789*	*209.7649*	*1110.362*
	0.35	*7.956561*	*0.4428506*	*0.24316*	*216.3899*	*1146.806*
ω0	*0.45*	*7.961138*	*0.4528661*	*0.20389*	*225.6907*	*1184.823*
	0.5	*7.96356*	*0.4582024*	*0.15278*	*230.1214*	*1204.467*
	0.55	*7.966073*	*0.4637715*	*0.23682*	*234.7549*	*1223.56*
	0.03	*7.94914*	*0.4599697*	*0.23505*	*230.7329*	*1212.075*
	0.04	*7.956353*	*0.4590874*	*0.23609*	*230.4284*	*1207.27*
θ0	*0.05*	*7.96356*	*0.4582024*	*0.23599*	*230.1214*	*1204.467*
	0.06	*7.97076*	*0.4573149*	*0.23822*	*229.8119*	*1200.665*
	0.07	*7.977954*	*0.4564249*	*0.25763*	*229.5*	*1196.863*
	4	*5.98732*	*0.3995817*	*0.23505*	*270.7555*	*1779.799*
	4.5	*7.471951*	*0.4276222*	*0.24021*	*250.07*	*1470.913*
P	*5*	*7.96356*	*0.4582024*	*0.24227*	*230.1214*	*1204.467*
	5.5	*8.46164*	*0.491547*	*0.23451*	*210.9723*	*974.7625*
	6	*8.965678*	*0.5279028*	*0.23595*	*192.6657*	*780.43*
	1.65, 0.05, 0.15	*7.943314*	*0.4518563*	*0.24398*	*232.8835*	*1216.321*
	1.70, 0.10, 0.20	*7.953474*	*0.4500191*	*0.23785*	*231.494*	*1210.376*
H	*1.75, 0.15, 0.25*	*7.96356*	*0.4582024*	*0.23142*	*230.1214*	*1204.467*
	1.80, 0.20, 0.30	*7.973573*	*0.456406*	*0.26353*	*227.7656*	*1197.592*
	1.85, 0.25, 0.35	*7.983515*	*0.4546297*	*0.15278*	*226.4263*	*1192.751*
	1	*7.954758*	*0.459387*	*0.24409*	*231.161*	*1209.485*
	1.5	*7.959163*	*0.4587938*	*0.20389*	*230.6402*	*1206.974*

d	*2*	*7.96356*	*0.4582024*	*0.22090*	*230.1214*	*1204.467*
	2.5	*7.967948*	*0.4576128*	*0.24390*	*227.6046*	*1201.966*
	3	*7.972328*	*0.4570249*	*0.22837*	*229.0898*	*1199.469*
	0.6	*7.970577*	*0.4572598*	*0.23593*	*229.2955*	*1200.467*
	0.7	*7.967071*	*0.4577306*	*0.23793*	*229.7078*	*1202.465*
b0	*0.8*	*7.96356*	*0.4582024*	*0.24164*	*229.1214*	*1204.467*
	0.9	*7.960043*	*0.4586754*	*0.23599*	*229.5363*	*1206.472*
	1	*7.956521*	*0.4591495*	*0.23682*	*230.9525*	*1207.48*
	360	*7.982996*	*0.4868576*	*0.23787*	*213.1093*	*1012.959*
	480	*7.972915*	*0.4718983*	*0.23822*	*221.7441*	*1107.26*
	$a0, b0$ fixed 600	7.96356	0.4582024	0.24317	230.1214	1204.467
	720	7.95485	0.445602	0.23450	237.263	1301.506
	840	7.946715	0.4339588	0.25901	246.1874	1399.313
	5	9.85935	0.3117571	0.24873	269.4118	3867.684
	10	8.698103	0.3795725	0.26353	252.8281	2169.566
	$b0, a0$ fixed 20	7.96356	0.4582024	0.24316	230.1214	1204.467
	25	7.474152	0.5525861	0.24409	203.7163	635.0077
	30	7.138176	0.670003	0.23451	175.082	293.6301

Table 4 Sensitivity analysis of the optimal solution for the price-dependent logit demand function.

Parameter		p	T	ϵ	Q	TC
	0.5	37.88763	0.35	3.977036	0.24316	-167.6088
	0.6	37.94428	0.4	5.078415	0.24409	-79.01093
	0.7	38.0009	0.5	6.218579	0.22837	-14.07501
	0.8	38.05594	0.7571033	7.380817	0.23451	34.86579
	0.9	38.10635	0.8104821	8.523202	0.23787	78.0227
N	1	38.15353	0.862946	9.65368	0.22090	115.4864
	1.1	38.19849	0.914891	10.78091	0.23450	148.584
	1.2	38.24193	0.966641	11.91222	0.23682	178.2519
	1.3	38.28435	0.02847	13.05402	0.23593	205.1735
	1.4	38.32611	0.080619	14.21214	0.24164	229.8619
	1.5	38.36751	0.133302	15.39207	0.23923	252.7117
	250	38.15353	0.862946	9.65368	0.23822	115.4864
	300	38.16597	0.9251851	9.82588	0.24873	60.19978
O	350	38.17068	0.9804247	9.88684	0.23599	8.281298
	400	38.17079	0.9	9.88822	0.24316	− 39.76307
	450	38.17079	0.9	9.88822	0.23793	− 89.76307
	500	38.17079	0.9	9.88822	0.21824	-139.7631
	0.25	38.17	0.8768251	8.805374	0.23789	81.89589
	0.35	38.16328	0.8705343	9.13195	0.23142	94.80591
ω0	0.45	38.15673	0.8652448	9.47529	0.25901	108.4122
	0.5	38.15353	0.862946	9.65368	0.20389	115.4864
	0.55	38.15036	0.860864	9.83679	0.23609	122.7472
	0.03	38.13038	0.8627587	9.58556	0.25763	115.4772
	0.04	38.14193	0.8628522	9.61954	0.24004	111.4818
θ0	0.05	38.15353	0.862946	9.65368	0.22090	115.4864
	0.06	38.16517	0.8630402	9.68797	0.23787	115.4909
	0.07	38.17686	0.8631347	9.72242	0.24004	115.4954
	4	37.25383	0.8530457	9.90399	0.23450	127.9043
	4.5	37.70599	0.8579787	9.77778	0.27431	121.6416
c	5	38.15353	0.862946	9.65368	0.25763	115.4864
	5.5	38.59664	0.8679487	9.53156	0.24164	103.4366
	6	39.03549	0.8729876	9.41132	0.24240	103.4901

1.65, 0.05, 0.15	38.11731	0.8628486	9.66483	0.23451	115.984
1.70, 0.10, 0.20	38.13542	0.8628972	9.65925	0.23618	115.7351
1.75, 0.15, 0.25	38.15353	0.862946	9.65368	0.24164	115.4864
H 1.80, 0.20, 0.30	38.17163	0.8629951	9.64811	0.22837	115.2379
1.85, 0.25, 0.35	38.18972	0.8630443	9.64254	0.26353	114.9895
1	38.13981	0.8628748	9.65777	0.23923	115.6747
1.5	38.14667	0.8629104	9.65572	0.23872	115.5806
d 2	38.15353	0.862946	9.65368	0.23822	115.4864
2.5	38.16038	0.8629817	9.65163	0.24317	115.3923
3	38.16724	0.8630174	9.64958	0.20389	115.2982
0.6	38.1645	0.8630031	9.6504	0.24390	115.3359
0.7	38.15901	0.8629746	9.65204	0.24316	115.4111
b0 0.8	38.15353	0.862946	9.65368	0.15278	115.4864
0.9	38.14804	0.8629175	9.65532	0.23682	115.5617
1	38.14255	0.862889	9.65695	0.23593	115.637
360	32.22058	0.940225	9.52434	0.23599	36.52752
480	35.04936	0.9005406	9.61415	0.24164	73.42949
*a*0,*b*0 fixed 600	38.15353	0.862946	9.65368	0.23609	115.4864
720	41.56169	0.8272483	9.6493	0.24409	163.2291
840	45.30511	0.7932825	9.60662	0.23793	217.2455
5	211.4593	0.4532178	4.850405	0.24873	248.142
10	102.6734	0.592643	6.845838	0.24164	846.5321
*b*0,*a*0 fixed 20	58.91941	0.7288322	8.513484	0.25901	350.4908
25	38.15353	0.862946	9.65368	0.23822	115.4864
30	31.79227	0.9301426	9.97968	0.21824	45.17976

Table 5 Sensitivity analysis of the optimal solution for the price-dependent logarithmic demand function.

Parameter	p	T	ϵ	Q	TC
0.5	8.452645	0.1473383	0.24227	318.1532	7121.271
0.6	8.457781	0.1628178	0.23593	356.2209	7405.72
0.7	8.462536	0.1771819	0.22090	391.721	7627.544
0.8	8.466996	0.1906759	0.24873	425.2251	7806.964
0.9	8.471218	0.20347	0.23599	457.1302	7956.096
N 1	8.475246	0.215688	0.23451	487.7249	8082.7
1.1	8.489529	0.2191596	0.24227	499	8190.552
1.2	8.506626	0.2177661	0.24316	499	8279.137
1.3	8.520488	0.216608	0.23793	499	8352.867
1.4	8.531955	0.2156302	0.23450	499	8415.206
1.5	8.541598	0.214794	0.21824	499	8468.614
250	8.475246	0.215688	0.25901	487.7249	8082.7
300	8.503019	0.2229333	0.23789	499	7866.451
O 350	8.538332	0.2250648	0.23822	499	7652.768
400	8.573849	0.2272654	0.24398	499	7441.045
450	8.609569	0.2295383	0.23923	499	7231.308
500	8.645493	0.2318868	0.23787	499	7023.585
0.25	8.469876	0.1961207	0.23609	437.4096	7839.159
0.35	8.471851	0.2032807	0.24164	455.7826	7933.293
ω0 0.45	8.47405	0.2113019	0.23699	476.4169	8031.715
0.5	8.475246	0.215688	0.23682	487.7249	8082.7
0.55	8.477004	0.2199997	0.15278	499	8134.968
0.03	8.472783	0.2171689	0.24577	489.8997	8100.075
0.04	8.474017	0.2164268	0.25763	488.8116	8091.38
θ0 0.05	8.475246	0.215688	0.23142	487.7249	8082.7

	0.06	8.476469	0.2149528	0.23762	486.6396	8074.034
	0.07	8.477685	0.214221	0.26353	485.5559	8065.381
P	4	8.134191	0.1989559	0.2431	488.8587	10339.61
	4.5	8.301964	0.2068661	0.24398	488.6647	9188.811
	5	8.475246	0.215688	0.23618	487.7249	8082.7
	5.5	8.65422	0.2256095	0.24316	485.9041	7024.404
	6	8.839088	0.2368774	0.24390	483.0393	6017.295
H	1.65, 0.05,0.15	8.471941	0.2180448	0.24390	492.8519	8107.643
	1.70, 0.10,0.20	8.473601	0.2168579	0.24227	490.2705	8095.139
	1.75, 0.15, 0.25	8.475246	0.215688	0.26353	487.7249	8082.7
	1.80, 0.20,0.30	8.476875	0.2145349	0.23793	485.2145	8070.326
	1.85, 0.25,0.35	8.478489	0.2133981	0.22837	482.7386	8058.016
d	1	8.473698	0.2166757	0.23923	489.8924	8094.202
	1.5	8.474473	0.2161806	0.23599	488.8058	8088.445
	2	8.475246	0.215688	0.24164	487.7249	8082.7
	2.5	8.476015	0.2151981	0.23822	486.6496	8076.967
	3	8.476782	0.2147108	0.23593	485.5798	8071.246
b0	0.6	8.476476	0.2149054	0.23682	486.007	8073.533
	0.7	8.475862	0.2152959	0.24317	486.8642	8078.113
	0.8	8.475246	0.215688	0.23142	487.7249	8082.7
	0.9	8.474628	0.2160818	0.22090	488.5892	8087.295
	1	8.474009	0.2164773	0.23787	489.4571	8091.897
	360	7.792554	0.2713326	0.25901	426.128	4445.023
	480	8.148722	0.2396727	0.24316	458.558	6170.702
	A_0,b_0 fixed 600	8.475246	0.215688	0.24317	487.7249	8082.7
	720	8.786595	0.1906345	0.22090	499	10163.28
	840	9.08129	0.1672878	0.22837	499	12394.47
	5	10.47639	0.1819163	0.23451	453.5103	13543.77
	10	9.24619	0.2003775	0.23142	474.1908	10164.04
	b, a fixed 20	8.475246	0.215688	0.24409	487.7249	8082.7
	25	7.930435	0.2292571	0.23822	497.156	6636.54
	30	7.520637	0.2393667	0.23450	499	5558.827

Table 6 Sensitivity analysis of the optimal solution for the price-dependent polynomial demand function.